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Fractal Dimensions In Perceptual Color Space: A Comparison Study Using Jackson Pollock'S Art

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Fractal dimensions in perceptual color space: A comparison study using Jackson Pollock's art

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The fractal dimensions of color-specific paint patterns in various Jackson Pollock paintings are calculated using a filtering process that models perceptual response to color differences ($L^*a^*b^*$ color space). The advantage of the $L^*a^*b^*$ space filtering method over traditional red-green-blue (RGB) spaces is that the former is a perceptually uniform (metric) space, leading to a more consistent definition of "perceptually different" colors. It is determined that the RGB filtering method underestimates the perceived fractal dimension of lighter-colored patterns but not of darker ones, if the same selection criteria is applied to each. Implications of the findings to Fechner's "principle of the aesthetic middle" and Berlyne's work on perception of complexity are discussed. © *2005 American Institute of Physics*. [DOI: 10.1063/1.2121947]

The use of fractal analysis to explain aesthetic properties of art is becoming a subject of great interdisciplinary interest to physicists, psychologists, and art theorists. Previous studies have addressed the classification of abstract expressionist art by the fractal dimension of the pigment patterns on the canvas as a method of artist authentication. Moreover, it has been proposed that the fractal structure of the pigment patterns is somehow connected to the aesthetic "value" of the painting. The patterns in question have traditionally been selected using filtering algorithms of red-green-blue (RGB) primaries, a perceptually nonuniform color space in which "distances" between perceptually just-differentiable colors are not the same for lighter and darker hues. Although RGB-based analyses have had success in devising categorization schemes for abstract paintings (see the cited literature), the use of this color space limits analyses that seek to cross compare the fractal dimension of different color patterns from a perceptual stance. The following summarizes the results of a fractal analysis performed on several paintings by the renowned artist Jackson Pollock, this time in a perceptually uniform color space that more closely replicates how the visual cortex would identify and differentiate individual colors. The data provide better insight into the fractal dimension and aesthetic nature of specific light and dark pigment patterns, and posit that the artist may have primarily used darker colors to engage the viewer.

I. FRACTALS IN ABSTRACT EXPRESSIONIST ART

Fractals are implicitly tied to the notions of chaos and irregularity,[1–3] and over the past 15 years have been increasingly associated with human perception issues. The problem of structure identification and discrimination in music, art, and visual processing has benefited greatly from this cross-disciplinary endeavor. For example, the authors of Refs. 4 and 5 pose the question of whether or not humans are "attuned" to the perception of fractal-like optical and auditory stimuli. Similarly, the results reported in Ref. 6 show that the quantitative accuracy of human memory possesses a fractal-like signature that can be measured in task repetition. Specifically, when subjects were asked to perform tasks such as repeatedly drawing lines of specific lengths or shapes, the statistical variations in the lengths have been shown to be not purely random noise, but fractally ordered "$1/f$" noise.

Recently, the use of fractal dimension analysis techniques for the study of paintings has become of interest,[7–12] which in the case of works by Jackson Pollock suggests that the fractal dimension of the paint patterns clusters suspiciously around the value $D_F \sim 1.7$. In Refs. 11 and 12, the analysis is extended to paintings by different artists, and addresses the full multifractal spectrum of the patterns. Furthermore, to overcome the problem of proper color choice (the focus of discussion in this paper), the notion of a *visual fractal* was introduced.[11] Instead of direct observation of colors, the focus instead shifted to edge structures. This is effectively an analysis of luminance gradients within the image, and not directly on the RGB color field distribution.

Implicitly related to this topic, the authors of Ref. 13 discuss the perceptibility of hierarchical structures in abstract or nonrepresentational constructs. In fact, rapid object recognition and categorization via boundary isolation versus "blob" identification is a subject of growing scientific interest (see Ref. 14 and related references therein). Similarly, the degree of complexity present in a scene is largely believed to be critical in maintaining the interest of an observer.[15,16] The fractal dimension is a natural measure of such complexity.

The predominant question remains: "Where is the fractal?" Does one calculate this statistic based on a pattern of a specific color? If so, how is this color selected and specified? A simple choice would be to pick the most abundant values of RGB primaries and digitally deconstruct the image to remove the appropriate matching pieces. Patterns which match this selection criteria can be called "physical colors," since

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the RGB primaries define the image as it appears (on the canvas).

However, the human visual processing system has evolved in such a way that the actual physical world is not always what is perceived by the brain. There is a long-standing argument addressing the questions of how we process scenes, what elements are important to a visual field, and so forth. As previously mentioned, the analysis in Refs. 11 and 12 studies the edge structure of paintings, based on the notion that we perceive contrast changes separately (or independently) from individual colors.

Similarly, perceived differences between colors themselves are nontrivial to quantify. In fact, use of RGB primaries for perceptual image analysis is flawed because the color space in question is not perceptually uniform. In this paper, previously reported fractal dimensions for various paintings by Jackson Pollock are recomputed using what will be termed "perceptual color selection," as opposed to physical color selection. The latter uses the simple RGB primaries, while the former involves computations in the *Commission Internationale de l'Eclairage* (CIE) $L^*a^*b^*$ color space.

The following will analyze six paintings by Jackson Pollock by determining the fractal dimension of specific patterns formed in the $L^*a^*b^*$ color space. These data will be compared to the fractal dimensions of the same color patterns in the usual RGB color space, and thus the results can be understood to represent the perceptual distinctions of colors on the canvas.

II. THE BASICS OF PERCEPTION

Before attacking the problem of detecting visual fractals, a brief primer on color vision and perception is in order. In fact, it was physicists who had the first major say in the foundations of this science, known in the literature as "psychophysics."

In the early 1800s, the trichromacy theory of vision was postulated by Young, and was later expanded upon by Helmholtz and Maxwell (later dubbed the Young-Helmholtz theory, much to the dismay of Maxwell).[17] The assertion was that color vision is the result of simultaneous stimulation of three photoreceptors in the eye, based on the RGB primary breakdown. Physiological confirmation of this hypothesis did not come until the 1960s, when three distinct cellular receptors in the eye (cones) were discovered to have peak sensitivities to light of $\lambda = 440$ nm (blue), 540 nm (green), and 580 nm (actually more yellow than red).

Meanwhile, the late 1800s saw the emergence of Hering's Opponent Theory of Vision.[17] Instead of a trichromatic basis for vision, Hering proposed that the perception of colors was derived from the contrasting of opposite color/intensity pairs: red-green, yellow-blue, and light-dark. Again, experimental physiological evidence for such a mechanism was revealed in the 1950s. In this case, two chromatic signals and a third achromatic one were detected in the optical nerve under various stimulation experiments.

Note that, unlike the trichromacy theory, the Opponent theory allows for object recognition based on luminosity or hue gradients alone, and hence no explicit color information is required. So, while the raw color stimuli may be perceived, it may not be this information which is transmitted to the visual cortex for eventual processing.

Most modern theories of color perception tend to constitute a mixture of the two aforementioned postulates in some fashion. This, of course, leads to the immediate question: is there a preferential order for object and color detection? Is one a primary mechanism, and the other secondary? Or, are they mutually independent processes that serve to provide diverse information about the scene considered? There is still no clear answer to these musings, although much work has been devoted to such studies (see texts such as, e.g., Ref. 18 and references therein for further reading).

III. CIE COLOR SYSTEMS

The *Commission Internationale de l'Eclairage*, or CIE as it is more often known, was formed in an attempt to address and standardize the myriad aspects of color definition, reproduction, and perception via a rigorous set of mathematical standards and transformations. Since actual color perception can vary depending on the external conditions (ambient lighting) and internal conditions of the observer (neurophysiology of vision mechanism), a set of "invariant" standards is useful in describing ideal conditions under which observations and comparisons can be made.

In order to establish consistent external lighting variables, the CIE defined the standard illuminants to be those conditions which represent the complete spectral power distribution of a particular state. The most widely used of these standards are the D illuminants, which characterize the conditions of "average daylight." In the present work, all CIE conversions will reference the D65 illuminant, which corresponds to standard average daylight with a spectral temperature of 6500 K.[17,19] Note that the D-illuminants standards cannot be reproduced by any known physical source of light. Conversely, the earlier A-, B-, and C-illuminants were based on the spectral power distributions of (filtered) incandescent tungsten light (2854 K).[17] This mild lack of chromatic reproducibility is an inherent problem with digital analyses of images; however, with a 24-bit color system it is doubtful that it constitutes a large concern.

It should be noted that CIE color systems are primarily designed for industrial (textile) color-matching and color-gamut consistency in color displays. While many of their intricacies are based on human perception principles, they are not meant to fully represent the neural processes that occur in vision. For the purposes of this paper, however, they are certainly a good first-pass approach to the problem.

IV. FILTERING VISUAL FRACTALS

To date, the color-filter process has relied on the fact that the target colors are the mixture of RGB triplets. Such a color basis is certainly not unreasonable, and in fact forms a large base of the tristimulus theory of color vision. However, further inspection of color theory reveals that the three-dimensional RGB space is not perceptually uniform. That is, two colors that are a fixed distance β_{RGB} away from a base stimulus may not be equally different from a perceptual stance.

A. Alternate color representations

Furthermore, the RGB specification is deficient in the sense that, as an additive color scheme, it cannot reproduce all observed colors. In 1931, the CIE set out to formulate an accurate color space. Known as the CIE XYZ space, these tristimulus primaries themselves are not visible in the same sense as R, G, and B, but are rather an "imaginary" basis introduced to allow for reproduction of all observable colors. Specific colors $C(\lambda)$ are matched by combining appropriate amounts of red, green, and blue primaries (denoted r, g, and b). However, in many cases, it was noted that perfect matches could not be made in such a fashion. Instead, one could match combinations of two of the three primaries with a suitable combination of the target color and the third primary. Arithmetically, this implies

$$C(\lambda) + r\mathbf{R} = b\mathbf{B} + g\mathbf{G}, \tag{1}$$

and so the target $C(\lambda)$ is formed by a negative contribution from one of the primaries. The CIE XYZ system thus reproduces the entire spectrum of observable colors.

For a standard D65 illuminant observer, the transformation is a simple linear one of the form

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 0.412\,424 & 0.212\,656 & 0.019\,332\,4 \\ 0.357\,579 & 0.715\,158 & 0.119\,193 \\ 0.180\,464 & 0.072\,185\,6 & 0.950\,444 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}, \tag{2}$$

with the inverse transform yielding negative coefficients, as indicated above. The exact form of the matrix in Eq. (2) is somewhat dependent on the color gamut and standard white being used for display purposes. In the case of this paper, the matrix values are for the sRGB color scheme (for "standard RGB"), and will primarily be adopted for the analysis herein. However, comparison with other transformation schemes will be discussed.

Unfortunately, while the XYZ space is more physically realistic in terms of color reproducibility, it is still not perceptually uniform. The CIE addressed these issues, and offered several solutions as recently as 1976.

B. CIE-$L^*a^*b^*$ space: Perceptual uniformity

A truly perceptually uniform space, the CIE-$L^*a^*b^*$ color space is a nonlinear transformation of the XYZ space

$$\begin{aligned} L^* &= 116\, f(Y/Y_0) - 116, \\ a^* &= 500[f(X/X_0) - f(Y/Y_0)], \\ b^* &= 200[f(Y/Y_0) - f(Z/Z_0)], \end{aligned} \tag{3}$$

where $f(X/X_0) = (X/X_0)^{1/3}$ if $(X/X_0) > 0.008\,856$, and $f(X/X_0) = 7.787(X/X_0) + 16/116$ otherwise.[17] Here, the values $(X_0, Y_0, Z_0) = (0.3127, 0.3290, 0.3583)$ are the standard (white) tristimulus values for a 2° observer in the D65 illuminant (in general, one can make the approximation $X_0 = Y_0 = Z_0 = 1/3$). The coordinate L^* represents the perceived luminosity, and covers the range of luminance scales (0 being black, 100 being white). The remaining coordinates a^* and b^* are the relative red-green and blue-yellow content, analogous to Hering's Color Opponent theory and more realistic ocular color detection processes.[17]

The perceptual color difference is then the Euclidean distance in $L^*a^*b^*$ space,

$$\beta_{L^*a^*b^*} = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2}. \tag{4}$$

One immediately notes from the form of Eq. (4) that the structures of the RGB and $L^*a^*b^*$ color spaces are quite different. This suggests that the relative structures obtained by color-filter processes are largely dependent on the color-matching system at hand. Specifically, one might expect that the patterns selected by RGB filtering criteria do not conform to those of an $L^*a^*b^*$ filter. That is, the physical distribution of like colors may not correspond to the perceived distribution of colors. If the structures are sufficiently different, then this can weaken arguments that suggest that patterns of specific fractal dimension are pleasing to observers.

The difference in measured spectra may indeed be a visual effect, if the eye functions on a similar uniform "cutoff" level for like-color discrimination. However, the actual color information of the system may not be the most important contributor to first-order visual processing systems.

V. ANALYSIS AND RESULTS

The images analyzed herein are digital scans at 300 dpi, with side lengths ranging from 1000–2000 pixels. In this case, each pixel corresponds to a length scale on the order of a few tenths of a centimeter, corresponding to a target $L^*a^*b^*$ color (within an allowed color radius); they are filtered to form a "perceived" representation of a particular pattern. The fractal dimension of the resulting pattern is determined by the traditional box-counting technique, where the covering boxes range in size from $d = 1024$ px to $d = 4$ px, or length scales of roughly 1.5–2.5 m to a few millimeters. The box-counting analysis thus covers about three orders of magnitude.

The calculated fractal dimensions D_F for both RGB and $L^*a^*b^*$ spaces are displayed in Table I. What is immediately apparently and interesting to note is that $L^*a^*b^*$ space is much more sensitive to changes in lighter colors, implying that the calculated dimensions for cream or white blobs with equal β in RGB space will in general *not* be the same in the perceptually uniform space. This suggests that the overall structure of the blobs may depend on the individual who perceives them, and hence the structures may be perceptually different than their physical color distribution (RGB space) suggests. Figures 1 and 2 demonstrate how the physical RGB distribution of a light color is significantly less than the perceptual $L^*a^*b^*$ distribution for the same color.

In fact, for an equal value of $\beta_{L^*a^*b^*}$, the values of D_F in $L^*a^*b^*$ space for lighter colors are consistently higher than

TABLE I. Comparison of fractal dimensions calculated by RGB and $L^*a^*b^*$ filtering processes for two different RGB-XYZ transformations (D65 illuminants). The radii in $L^*a^*b^*$ color space are chosen to produce approximately the same value of D_F for darker colors (in this case, $\beta_{L^*a^*b^*}=15$). The number in parentheses is the error in the least-square fit used to calculate the fractal dimension.

Color ID	D_F (RGB)	D_F ($L^*a^*b^*$; sRGB D65)	D_F (Adobe RGB D65)
		Reflections of the Big Dipper (1947)	
Black	1.77	1.78 (0.04)	1.77 (0.04)
Yellow	1.35	1.53 (0.08)	1.70 (0.06)
		Number One A 1948	
Black	1.77	1.78 (0.03)	1.76 (0.04)
White	1.57	1.79 (0.04)	1.81 (0.03)
		Undulating Paths	
Black	1.76	1.75 (0.05)	1.75 (0.05)
Yellow	1.56	1.79 (0.04)	1.80 (0.04)
		Number One 1949	
Gray	1.73	1.82 (0.03)	1.83 (0.03)
Yellow-gray	1.71	1.83 (0.03)	1.84 (0.03)
		Blue Poles (1952)	
Black	1.74	1.49 (0.07)	1.52 (0.07)
Gray	1.68	1.78 (0.02)	1.79 (0.03)
		Autumn Rhythm (1950)	
Black	1.70	1.54 (0.05)	1.51 (0.05)
White	1.30	1.59 (0.04)	1.64 (0.03)

the equivalent values in RGB space (for fixed β_{RGB}). This result is justifiable based on the nature of the perceptual uniformity of $L^*a^*b^*$ space. In traditional RGB spaces, lighter colors occupy a much larger volume than darker colors. Thus, an analysis that uses a color radius β_{RGB} will miss significant portions of the space, and will filter a pattern having a shallower range of "undistinguishable colors." The transformation to $L^*a^*b^*$ space shrinks the volume of the lighter colors (which correspond to higher luminosity values); thus, the associated analysis will include a much richer depth of colors (and hence a larger pattern will result). An interesting "test" of such perceptual distinction of patterns would be to study the differences in fractal dimensions calculated from paintings by different artists who largely use subtle, nonluminous colors.

In many cases, the former light color dimensions surpass the D_F for the darker colors, whereas before they were less than or equal to them. If it is true that a viewer will have a preference for midrange values of the fractal dimension, $D_F \sim 1.3$–1.7 (as suggested by the principle of aesthetic middle[21] and also supported by recent data from Ref. 9), then it can be inferred that the darker patterns "fix" the fractal dimension for the whole painting. This is a similar conclusion to that observed in painting "construction" by Taylor *et al.*,[8] who dubbed this the "anchor layer."

The color spaces used in this analysis correspond to average, human color receptor responses. Individual variations in these responses, as well as those who possess color deficiencies (color blindness), could certainly impact the perceived dimensionality of the patterns. Indeed, it might be that the artist himself did not "see" the same pattern as his audience did. However, color-blindness conditions are more a function of decreased color hue sensitivity, rather than luminosity perception (which is the dominant channel in $L^*a^*b^*$ space). Further studies could address these perceptual differences.

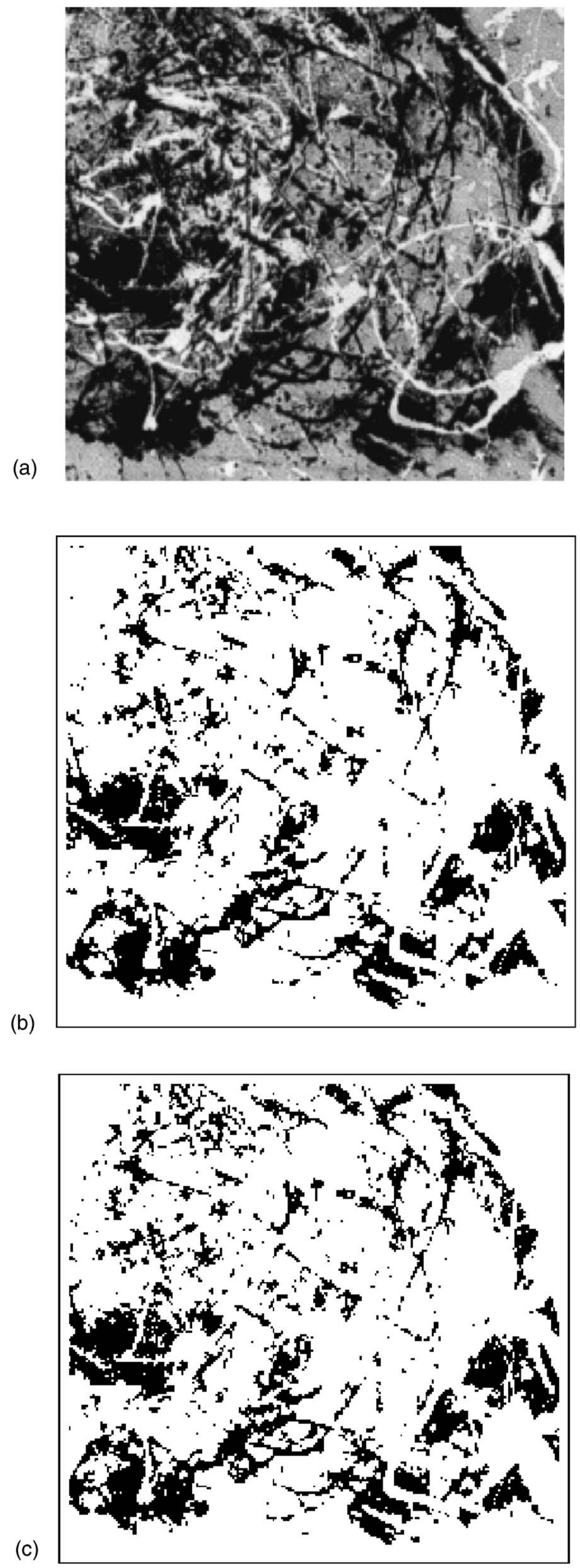

FIG. 1. Portion of black pigment filter of *Autumn Rhythm* showing (a) raw image, (b) physical RGB distribution, and (c) perceptual $L^*a^*b^*$ distribution.

As a result, these conclusions can thus be thought of as a

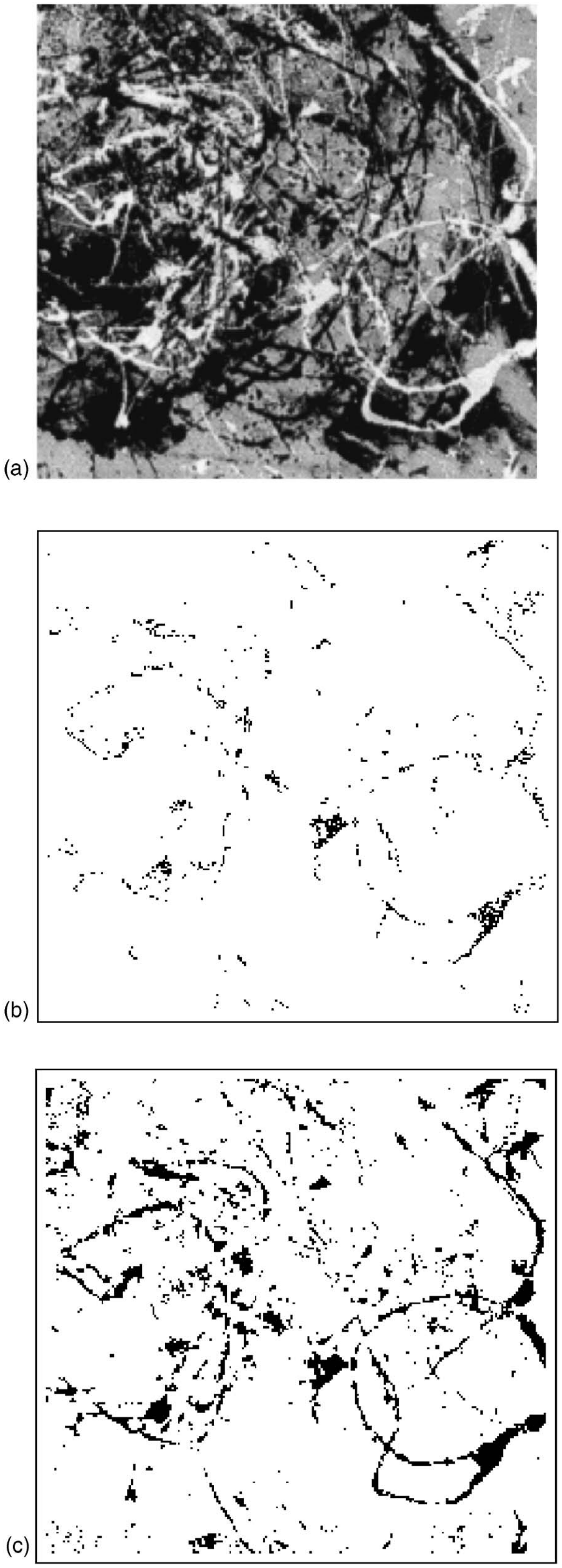

FIG. 2. Portion of white pigment filter of *Autumn Rhythm* showing (a) raw image, (b) physical RGB distribution, and (c) perceptual $L^*a^*b^*$ distribution corresponding to the data in Table I.

preliminary assessment of perceptual color fractals. Further experimentation, complemented by psychological behavioral data, is certainly required before definite conclusions can be drawn.

Choice of color scheme and illuminant

As previously mentioned, there are numerous possible choices of RGB-XYZ transformation matrices used in Eq. (2). These depend on the color system being used (e.g., NTSC, PAL), the palette adopted by computer monitors, and ultimately the standard white defined by the illuminant. Table I offers a comparison to another D65 illuminant transformation labeled "Adobe RGB-XYZ," having components

$$\begin{pmatrix} 0.576\,700 & 0.297\,361 & 0.027\,032\,8 \\ 0.185\,556 & 0.627\,355 & 0.070\,687\,9 \\ 0.188\,212 & 0.075\,284\,7 & 0.991\,248 \end{pmatrix}. \tag{5}$$

It is clear from the results that the choice of scheme is mostly inconsequential to the dimensions being calculated. Discrepancies can be noted in few of the color patterns considered. In fact, these could be explained away as an improper choice of RGB primaries to begin with. This cross comparison could in fact be used as a method for determining the "actual" RGB coordinates required for the analysis. In any event, the conclusions from the previous section are still supported: For a fixed color space radius, lighter-colored patterns will have a perceptually higher fractal dimension than darker ones.

VI. DISCUSSION AND CONCLUSIONS

Calculating the fractal dimension of patterns based on their RGB coordinates in the digital representation is not reflective of visual selection criteria for the same colors due to the nonmetric nature of the space. The $L^*a^*b^*$ color space is a more natural choice that reflects the color response of the human perception system, and is a consistent metric space. This study has suggested that, if the fractal dimensions for dark patterns are in agreement with previous analysis methods (which they should be, since the color spaces for darker colors overlap fairly closely), then the lighter-colored patterns possess a much higher fractal dimension approaching $D_F=2$. This implies that the distribution of lighter colors—having higher complexity—would saturate the visual system.

These results can be related to Fechner's "principle of the aesthetic middle," which states that a viewer will tolerate for the longest period of time a visual scene of moderate complexity.[21] This was experimentally verified by Berlyne[15,16] for statistical distributions, and more recently applied to fractal analysis by Taylor.[9,10] The latter reported that human preference for fractals of dimension $D\sim 1.3$ is the highest.

However, this work has found that the dimensions for the color patterns are significantly above the "aesthetic middle" dimension of 1.3. What then are the motivations for painting patterns that specifically are *not* aesthetically pleasing to the average viewer? This is currently an open question that has no single satisfactory answer. Borrowing again from the field of aesthetic research, it is possible to explain Pollock's choice of dimensions by appealing to the peak shift effect, one of the "eight laws of artistic experience."[22] The peak shift effect is an experimentally verified cognitive phenomenon in which visual interest or identification is strengthened by overtly enhancing key characteristics of an object or image (such as the "larger-than-life" features of caricatures in political cartoons). These enhanced characteristics are explicitly *not* aesthetically pleasing, but their purpose is to grab attention and convey key recognition information in a rapid fashion (see Ref. 23 for a detailed discussion).

Alternatively, the relevance to the present work can be understood by considering the relative difference in fractal dimensions between perceptual colors in Pollock's work. That is, based on the notion that lowest fractal dimensions are more appealing to observers, this indicates that it is primarily the darker patterns that play a role in capturing the interest of the observer. This is consistent with Taylor's earlier notion of the anchor layer, and in fact serves as a method of "identifying" the most salient pattern on the canvas. In fact, the "attractiveness" of the pattern (based on lower fractal dimension) and the assertions of this paper could be experimentally verified through eye saccade-type or other subject perception experiments.

One could speculate that Pollock deliberately "tuned" his paintings to contain these color visual structures, based on an intuitive understanding of the visual arts and aesthetics. This would then indicate a third level of structure in his paintings, in addition to the physical fractals of the paint blobs, as well as the edge fractals created by the luminosity gradients of overlapping pigments.[12] If this is indeed true, then it further exemplifies the artistic genius that he demonstrated in creating visually complex, yet emotionally compelling, nonrepresentational scenes.

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