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Static 2-D Solution of a Mooring Line of Arbitrary Composition in the Vertical and Horizontal Operating Modes

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STATIC 2-D SOLUTION OF A MOORING LINE OF ARBITRARY COMPOSITION IN THE VERTICAL AND HORIZONTAL OPERATING MODES

by

B.W. Oppenheim[1] and P.A. Wilson[2]

Synopsis

A theoretical static solution is reported for a two-dimensional mooring line having an arbitrary multi-segment composition of wires, chains and synthetic ropes, and buoys. The segments can be buoyant, neutrally buoyant or heavier than water, and they can be nonlinearly stretchable. The buoys can be of constant positive and negative buoyancy, thus representing submerged buoys and hung weights. A mooring line can operate in the 'horizontal' or 'slack' mode, and in the 'vertical' or 'tension-moor' mode. In the former case, a surface-floating 'spring-buoy' is allowed, as well as a linear slope of the sea bottom at the anchor.

Introduction

Presented here is a static solution to the classical marine mooring problem, where given is the mooring line material composition, water depth, slope of the sea bottom and the vessel geometry, and desired are the line shape and loads as functions of the horizontal restoring force in the line. It is assumed that the entire line lies in the vertical plane, i.e. the problem is two-dimensional. This assumption is valid in the static domain if there is no current load and no friction on the sea bottom in the direction perpendicular to the line plane. The latter condition becomes redundant if the entire line is suspended, i.e. no portion of it lies on the bottom, except for the point of attachment to the anchor.

This problem has been addressed many times in the literature and there exist literally hundreds of solutions for various specific cases. In the present paper the emphasis is placed on the arbitrary composition of the line, and on the arbitrary mode of the line operation. This solution is valid for a multi-segment mooring line with each of the segments being of different positive or negative buoyancy, or being weightless in water. Typically, the wires and chains are of positive weight while the recently introduced synthetic ropes are either neutrally buoyant or slightly buoyant, or slightly heavier than water. With regard to the segment elasticity, the segments can be linearly stretchable, as is typically the case with wires and chains, or they can be nonlinearly stretchable, as is the case with the synthetic ropes. The nonlinear stress-strain relationship is usually approximated by an exponential law. In fact, it is the line axial elasticity that contributes the most to the mooring restoring properties in these ropes, while the wires and chains provide the spring effect mostly through the catenary mechanism. There can be any mix of the above segments in a line. The neighbouring segments can be connected directly to each other or through a shackle or other hung weight, or through a submerged buoy of constant buoyancy. The hung weights and buoys are assumed to be dimensionless.

A mooring line holding the vessel in position (either on its own, or as part of a multi-line system) can be said to operate either in the 'horizontal' mode (also called the catenery or slack mode), or in the 'vertical' mode (which is also called the tension-moor mode). These expressions are somewhat vague, in fact it will be shown that there is a continuous transition between the two modes which results in a single mathematical model being applicable to both of them.

In the horizontal mode, the line extends between the anchor (typically of a fluke type) on the bottom and the vessel fairlead over a considerable horizontal distance. This distance is often equal to several times the water depth. The horizontal restoring force is provided primarily by the line itself, i.e. by its catenary and elastic properties. Frequently the line is attached to a surface-floating buoy rather than to the vessel itself, and only a short rope connects the buoy to the vessel proper. This is done for two reasons.

Firstly, it increases the mooring restoring properties, due to the coupling between the horizontal force in the line and the vertical restoring buoyancy force of the buoy.

Secondly, such a buoy facilitates the line deployment and maintenance. The present solution is valid when the buoy is present. In such a case, the line is defined as extending from the anchor to the vessel fairlead, and the buoy is considered as an integral part of the line.

A mooring line operating in the horizontal mode frequently has a long portion of it lying on the sea bottom. In fact, the longer is that portion, the safer is the mooring. The vessel held in this mode is assumed to float at a constant draft, trim and heel.

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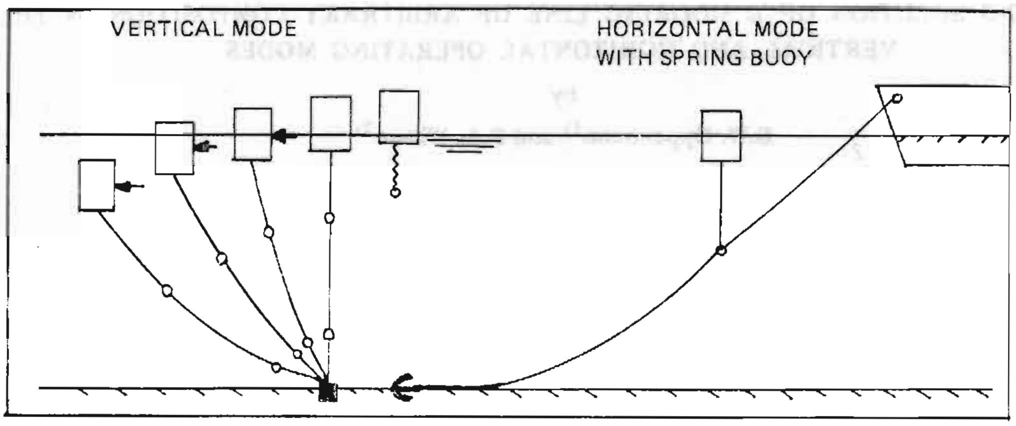

Figure 1. Vertical and horizontal modes of mooring line.

A mooring line in the vertical mode is entirely suspended in water between the anchor and the vessel; It can be totally vertical, or almost vertical with some residual catenary effect present. The anchor is typically of the gravity type (eg. a big chunk of concrete buried in the bottom). The vessel is held by the line at the draft somewhat greater than that of the freely-floating vessel, and the excess buoyancy keeps the line in tension at all times. Furthermore, as the vessel moves away from the equilibrium position, the line pulls it deeper into the water, and the increased buoyancy acts as a restoring force, together with the line elasticity and the residual catenary effect. Of course, no spring buoy is applicable in this mode.

Figure 1 illustrates the two modes of operations. It is evident that the shape of the line alone is not sufficient to differentiate between the two modes, except for the case of a vertical line which obviously belongs to the vertical mode. The classification adopted here for identifying the modes is based on the vertical span of the mooring line. The vertical mode is defined as that in which the line vertical span varies with the horizontal restoring force, i.e. the line shape and loads are coupled with the buoyancy of the vessel.

In the horizontal mode, the vertical span of the line is assumed to be constant, i.e. the vessel floats at constant draft, trim and heel. However, the draft variations of the spring buoy (if any) with the horizontal tension are taken into considerations.

Theory

Figure 2 illustrates a coordinate system $X-Y$ utilized globally with the origin at the anchor. Also shown is a local system x_i-y_i with the origin at the lower end of the i-th segment. Let there be N segments, $(N > 1)$ and let the first segment be latched onto the anchor and the N-th onto the vessel fairlead. Each segment is described by the following six parameters:

- w_i unit weight in water of the inelastic segment ($w_i < 0$ for a buoyant segment),
- G_i concentrated hung weight in water suspended at the top of the segment, ($G_i < 0$ denotes the net buoyancy of a submerged buoy),
- s_i unstretched length,
- F_i breaking (or proof) load,
- p_i, q_i two constants defining the axial stress-strain relationship:

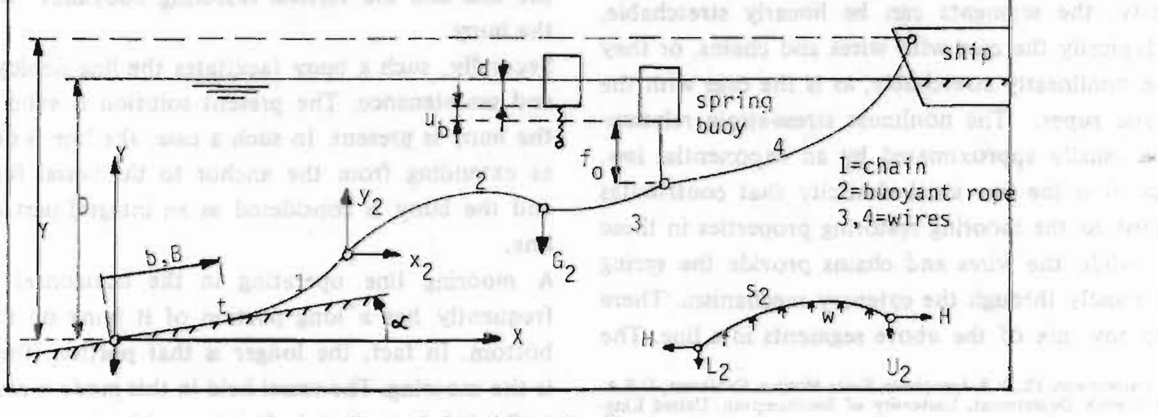

Figure 2. Sample mooring line.

$$\frac{\Delta s_i}{s_i} = p_i \left(\frac{T}{F_i}\right)^{q_i} \tag{1}$$

where T is the axial load. The flexural rigidity is neglected.

The derivations are organised into four steps, as follows.

1) The elementary catenary equations are reviewed for an arbitrary segment of a mooring line with no axial elasticity.
2) Corrections for the axial stretch of a segment are derived for the cases of the suspended and resting-on-bottom segments. Considered are the nonlinear and linear stretch.
3) The vertical mode solution is derived for both non-vertical and vertical elastic lines.
4) A derivation is given of the horizontal mode solution, again for an elastic line. Both the totally suspended and the slack lines are considered.

1. General equations for inelastic segment

The catenary equations for an i-th segment are shown in the Appendix to be

$$\begin{aligned} x_i &= \frac{H}{w_i} ln\ [(U_i + \sqrt{U_i^2 + H^2})/(L_i + \sqrt{L_i^2 + H^2})] \\ y_i &= \frac{1}{w_i}\ [\sqrt{U_i^2 + H^2} - \sqrt{L_i^2 + H^2}\,] \end{aligned} \tag{2}$$

where H, U_i and L_i are respectively the horizontal tension (constant throughout the line), and the vertical tensions at the segment upper and lower ends. Equations (2) are limited to the catenary shapes only, i.e. $w_i \neq 0$. When the segment is weightless, ($w_i = 0$), it is straight when loaded in tension, and its spans are then

$$\begin{aligned} x_i &= H \cdot s_i / \sqrt{U_i^2 + H^2} = H \cdot s_i / \sqrt{L_i^2 + H^2} \\ y_i &= U_i s_i / \sqrt{U_i^2 + H^2} = L_i s_i / \sqrt{L_i^2 + H^2} \end{aligned} \tag{3}$$

The sign convention utilized in equations (2) and (3) is that U_i is positive when pulling the segment upward and L_i is positive when pulling the segment downward.

Let the touch-down point of the mooring line be denoted by 't', not necessarely coinciding with the end of any segment. The unstretched length of the line on the sea bottom, b, is then

$$b = \sum_{i=1}^{t} b_i = \sum_{i=1}^{t} s_i \tag{4}$$

The vertical forces at the ends of the segment are obtained by summing up the line weight from t upwards,

$$L_i = \sum_{j=t}^{i-1} (s_j w_j + G_j) + R$$

$$U_i = L_i + G_i + s_i w_i = \sum_{j=t}^{i} (s_j w_j + G_j) + R \qquad i \neq 1 \tag{5}$$

where R is the vertical reaction at anchor, positive when the anchor pulls the line downward. The vertical tension acting at the touch-down point can take several values, depending whether the line is suspended, or tangent to the horizontal at the anchor, or resting on the sea bottom near the anchor. The possible values are:

0	if the segment becomes horizontal at anchor	
$-R$	if $i = 1$ and the entire line is suspended	(6)
$-H\tan\alpha = -R$	if the line is slack, with the part of it of the length b resting on the sea bottom. The slope of the bottom at the anchor is α.	

Equations (2)–(6) constitute the basis for derivations of both the vertical and horizontal modes of the inelastic line operations.

2. General equations of elastic segment

Nonlinear stretch

Let X_i, Y_i, W_i, S_i and B denote the stretched values of x_i, y_i, w_i, s_i and b, respectively.

The elongation of the i-th segment being in a catenary shape is, from (1)

$$\Delta s_i = p_i \int_i \left[\frac{T(s)}{F_i}\right]^{q_i} ds \tag{7}$$

where the integral extends over the suspended length of the segment. The axial tension varies along the segment,

$$T_i(s) = \sqrt{H^2 + V_i^2(s)} \tag{8}$$

where $V_i(s)$ is the vertical tension

$$V_i(s) = L_i + s w_i \tag{9}$$

Substituting (8) and (9) into (7) yields the general expression for the stretch,

$$\Delta s_i = p_i \int_i \left[\frac{\sqrt{H^2 + (L_i + s_i w_i)^2}}{F_i}\right]^{q_i} ds \tag{10}$$

In general, $q_i \in (0.1]$, therefore (10) must be evaluated numerically.

It is convenient to introduce the stretch factor C_i,

$$C_i = \frac{S_i}{s_i} = 1 + \frac{\Delta s_i}{s_i} \tag{11}$$

The total weight of the elastic segment is the same as that of the inelastic one,

$$W\,dS = w\,ds \tag{12}$$

therefore, from (11)

$$\frac{1}{W_i} = \frac{S_i}{s_i w_i} = \frac{C_i}{w_i} \tag{13}$$

The consequence of (12) is that L_i and U_i in equation (5) remain the same for both elastic and inelastic segments. Therefore the only elasticity-dependent quantity in equation (2) is the unit segment weight, w_i. Replacing w_i with W_i in these equations and utilizing equation (13), yields the spans of the elastic segment

$$X_i = C_i x_i \qquad Y_i = C_i y_i \tag{14}$$

When the segment is resting on the sea bottom with constant slope α, the tension in the segment is constant,

$$T_i(s) = H/\cos\alpha \tag{15}$$

Substituting (15) into (7) yields

$$\Delta s_i = p_i s_i \left(\frac{H}{F_i \cos\alpha}\right)^{q_i} \tag{16}$$

The stretch factor of this segment (or of its bottom-resting part) is denoted by C_i^b, where

$$C_i^b = \left.\frac{s_i + \Delta s_i}{s_i}\right|_{\text{bottom}} = 1 + p_i \left(\frac{H}{F_i \cos\alpha}\right)^{q_i} \tag{17}$$

The unstretched line length on the bottom, equation (4), becomes in the elastic case

$$B = \sum_{i=1}^{t} s_i C_i^b \tag{18}$$

Linear stretch

When the segment is linearly stretchable, the exponential law, equation (1), reduces to Hook's law upon the substitution

$$p_i = \frac{F_i}{A_i E_i} \quad , \quad q_i = 1 \tag{19}$$

where A_i and E_i are the segment cross-sectional area and the Young modulus. Now, the integral (10) can be solved analytically. The elongation becomes

$$\Delta s_i = \frac{1}{A_i E_i} \int_i \sqrt{H^2 + (L_i + s w_i)^2}\, ds =$$

$$\frac{1}{2A_i E_i} \Big| (L_i + s w_i) \left[H^2 + (L_i + s w_i)^2\right]^{1/2}$$

$$\left. + H^2 \ln \frac{(L_i + s w_i) + \left[H^2 + (L_i + s w_i)^2\right]^{1/2}}{w_i} \right|_i$$

Substituting the segment ends for the integral limits,

$$L_i + s w_i = \begin{cases} L_i & \text{if } s = 0 \\ U_i & \text{if } s = s_i \end{cases}$$

the stretch takes the form

$$\Delta s_i = \frac{1}{2A_i E_i w_i} \left\{ U_i \sqrt{H^2 + U_i^2} - L_i \sqrt{H^2 + L_i^2} + H^2 \ln \frac{U_i + \sqrt{H^2 + U_i^2}}{L_i + \sqrt{H^2 + L_i^2}} \right\}$$

Denoting the quantity in brackets {} by Q_i, putting $\Delta s_i = S_i - s_i$ and rearranging terms yields the desired stretch factor C_i

$$C_i = \frac{S_i}{s_i} = 1 + Q_i / A_i E_i w_i s_i \geqslant 1 \tag{20}$$

When the segment (or a part of it) is resting on the sea bottom, the tension in the segment is constant. Then equation (15) applies again, together with equations (7) and (19), to yield the stretch factor C_i^b

$$C_i^b = p_i \left(\frac{H}{F_i \cos\alpha}\right)^{q_i} = \frac{H}{A_i E_i \cos\alpha} \tag{21}$$

In summary, the catenary equations for the elastic segment can be written as

$$\begin{aligned} X_i &= C_i x_i \\ Y_i &= C_i y_i \\ W_i &= w_i / C_i \\ S_i &= C_i s_i \\ B &= \sum_{i=1}^{t} B_i = \sum_{i=1}^{t} C_i^b s_i \end{aligned} \tag{22}$$

where x_i, y_i and s_i are given by equations (2)–(3). Therefore, once the stretch factors C_i^b and C_i, $i = 1, \ldots, N$ are found, the derivations for an elastic segment reduce to those of an inelastic one.

Finally, it is necessary to mention that when the segment is weightless, the axial tension in it is constant and the stretch is then given directly by equation (1).

3. Mooring line in vertical mode

In this mode, the problem is formulated as follows. Known are the water depth D, the mooring line composition, and a function $Z(u_b)$ which describes the excess buoyancy of the vessel versus the vessel excess draft. In this mode, $b = B = 0$ by definition. Desired are the stretched line shape and loads (X_i, Y_i, L_i, U_i, $i = 1, \ldots, N$), the reaction at anchor, R, and the excess draft u_b, all as functions of the horizontal restoring force H.

In this mode, the touch-down point t corresponds

to the anchor, ($t = i = 1$), and the upper end of the line is attached directly to the vessel. Consider first the case of the line being non-vertical, but in some near-vertical catenary configuration,

$$X = \sum_{i=1}^{N} X_i > 0 \text{ , small} \tag{23}$$

The equilibrium of the horizontal forces in the line is assured since the tension H is carried throughout the line. The equilibrium in the vertical direction is

$$Z(u_b) = U_N = \sum_{i=1}^{N} (s_i w_i + G_i) + R \tag{24}$$

and the horizontal and vertical spans of the lines are given by (23) and by

$$Y = \sum_{i=1}^{N} y_i C_i = D - u_b - d \tag{25}$$

where d is the vessel draft in the freely-floating configuration, and D is the water depth.

The above equations constitute a set of non-linear algebraic equations and they must be solved iteratively. The iterations are performed with the help of the following merit function,

$$f_y(H) = U_N(H) - Z(u_b) \rightarrow 0 \text{ , } R = \text{const} \tag{26}$$

Thus it is sufficient to iterate H until f_y becomes sufficiently small. The sequence of calculations is as follows:

- Assume $R > R_o$, where R_o is the anchor reaction to the line being vertical ($H = 0$)
- Assume small H
- Calculate $L_i, U_i, i = 1, \ldots N$ from (5), with $t = 1$
- Find x_i and y_i, $i = 1, \ldots N$ from (2) or (3), as applicable
- Find C_i from (11) or (20), as applicable
- Find X_i and Y_i from (22)
- Find u_b from (25)
- Find $Z(u_b)$ from (24)
- Find f_y from (26)
- Update H as required by the sign of f_y, and repeat until f_y is sufficiently small.

The last succesful iteration provides the desired solution for one value only of R. Generating a series of such solutions for a range of systematically varied R yields the complete solution for the elastic non vertical line in the vertical mode.

If the line is vertical,

$$y_i = s_i \text{ , } Y_i = S_i \text{ , } H \equiv 0$$

and the iterations are handled as follows:

- Assume $R > 0$
- Calculate the vertical loads from (5)
- Calculate the stretch factors from (11) or (20)
- Calculate the stretched lengths S_i from (22)
- Calculate the vessel excess draft u_b from (25)
- Calculate the vessel excess buoyancy from (24)
- Calculate the objective function as

$$f_y(R) = U_N(R) - Z(u_b) \rightarrow 0 \tag{27}$$

- Iterate R until f_y becomed sufficiently small. The lack of convergence indicates that the line is too short or too long for the given depth.

4. Mooring line in horizontal mode

The vessel draft becomes now constant and therefore the total vertical span of the line between the anchor and the fairlead is known too, Y. Let there be a spring buoy attached to the top of a K-th segment, $1 \leqslant K < N$. Known as before are $D, s_i, w_i, G_i, F_i, p_i, q_i, i, \ldots N$ and $Z(u_b)$ of the buoy. Desired are the same functions as before and the stretched length of the line resting on the bottom, B.

Equations (2) and (3) still hold. The line weight summations, equation (5), must now be performed separately for the line portions below and above the spring-buoy,

$$L_i = \sum_{j=t}^{i-1} (s_j w_j + G_j) + R$$
$$1 < i \leqslant K \tag{28}$$
$$U_i = L_i + G_i + s_i w_i = \sum_{j=t}^{i} (s_j w_j + G_j) + R$$

$$L_i = \sum_{j=K}^{i-1} (s_j w_j + G_j) + R - Z(u_b)$$
$$K < i \leqslant N \tag{29}$$
$$U_i = L_i + G_i + s_i w_i = \sum_{j=K}^{i} (s_j w_j + G_j) + R - Z(u_b)$$

where the buoy net draft is a function of the vertical span of the line portion below the buoy,

$$u_b = D - B \sin\alpha - \sum_{i=t}^{K} Y_i - d - f_o \tag{30}$$

where d represents now the buoy draft in the freely-floating configuration, and $f_o > 0$ is the vertical distance between the buoy bottom and the top of the K-th segment.

The total spans of the line become

$$X = B \cos\alpha + \sum_{i=t}^{N} X_i$$
$$\tag{31}$$
$$Y = B \sin\alpha + \sum_{i=t}^{N} Y_i$$

The above equations constitute again a set of nonlinear algebraic equations which must be solved iteratively. Three variables are chosen for starting the iterations: H, R and b, however the last two are related. When the line is entirely suspended, $R \neq 0$ and $b = 0$. When the line is partly resting on the bottom, $b > 0$ and $R = -H\cos\alpha$. Thus in the entire domain of H there are two

distinct regions, one with $b > 0$ and the other with $b = 0$. The iterations are handled as follows.

- Assume b (maximum value)
- Assume $H > 0$
- By definition, $R = -H\cos\alpha$
- Find vertical tensions for the segments below the buoy, from (28)
- Find the unstretched segment spans for the segments below the buoy, from (2) or (3)
- Find the stretch factors C_i from (11) or (20) for the segments below the buoy
- Find the stretch factors C_i^b from (17) or (21) for the segments on the bottom
- Find the stretched spans X_i, Y_i and the length on the bottom B, from (22)
- Find the net draft of the buoy, u_b, from (30), and the buoy net buoyancy, $Z(u_b)$
- Find the vertical tensions of the segments above the buoy, from (29)
- Find the unstretched spans of the segments above the buoy, from (2) or (3)
- Find the stretch factors C_i, for the segments above the buoy, from (11) or (20)
- Find the stretched spans of the segments above the buoy, from (22)
- Find the total spans of the line, from (31)
- Compute the merit function f_y from

$$f_y = Y - b\sin - \sum_{i=1}^{N} Y_i \to 0 \qquad (32)$$

and if it is not sufficiently small, update H and repeat.

The convergence is usually fast but it may not be uniform if the line contains a mix of the positively and negatively buoyant elements. The solution, if available, is valid for the assumed value of b only. Repeating the calculations for systematically varied values of b between the maximum and zero yields a systematic series of solutions with the values of H being automatically correct. The results are organized into a set of functions

$$X_i(H), Y_i(H), B(H), L_i(H), U_i(H), u_b(H) \text{ and}$$
$$R(H), \quad (i = 1, \ldots N) \qquad (33)$$

If no solution exists for any value of b, the line is too short for the given span Y. Then $b = 0$, and R is being assumed for the independent variable in the iterations, where the iterations are conducted similarly to those above. Now the solutions are obtained for a systematic series of R instead of b. The upper limit of R is that yielding the line axial load equal to the breaking strength. The results are again organized into the functions

$$X_i(H), Y_i(H), L_i(H), U_i(H), u_b(H), R(H) \text{ and}$$
$$b(H) = B(H) = 0 \qquad (34)$$

The two sets of functions, (33) and (34) constitute together a complete solution of the elastic line in the horizontal mode.

When no spring buoy is present, $K = N$, and equations (29) and (30) become redundant. The remaining derivations and iterations are the same as before.

5. Sample calculations

It is evident from above derivations that most of the steps required for the vertical and horizontal modes as well as for the linearly and nonlinearly stretchable segments are identical. Therefore a single code can be developed to handle all above cases.

The theory has been implemented in three versions: on a CPM(Z-80) microcomputer, and on the ICL-2970 and CDC-7600 computers.

Tables 1 and 2 illustrate two sample computations, respectively. The first case represents a 3-segment line in the horizontal mode, having a spring buoy attached on top of the second segment. The lowest segment is made of chain, the middle segment of a synthetic rope, and the highest segment (connecting the buoy to the ship) of wire. The first part of Table 1 presents the input parameters of the mooring line, buoy and the environment and the second part contains the results. Both should be self-explanatory.

The second case represents a single-segment vertical-mode mooring. Again, the two parts of Table 2 illustrate the input and output, respectively.

The run times (in seconds) for the two cases were as follows:

	Case 1	Case 2
CPM(Z-80) microcomputer	980 s	895 s
ICL-2970 computer	185 s	182 s
CDC-7600 computer	6.2 s	6.0 s

Table 1

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PROGRAM SEML (SINGLE ELASTIC MOORING LINE)

MODE(TENSION-LEG OR CATENARY ?):      CATENARY

UNITS : METRIC(TONNEF,METERS,CENTIMETERS,KG),

                 INPUT AND NOMENCLATURE

    SEGMENT NUMBER (FROM ANCHOR)              1           2           3
UNSTRETCHED SEGMENT LENGTH (FT. OR M.)=  3.00E  3    6.00E  2    1.00E  2
UNIT WEIGHT IN WATER (LB/FT OR KG/M)  =  1.00E  2    5.00E  0    2.00E  1
FIRST CONSTANT OF STRETCH EQUATION    =  1.00E -9    1.80E -1    1.20E -9
SECOND CONSTANT OF STRETCH EQUATION   =  1.00E  0    5.00E -1    1.00E  0
BREAKING STRENGTH (KIPS OR TONNEF)    =  3.00E  2    3.00E  2    3.30E  2
WET WEIGHT ATTACHED TO THE SEGMENT
    UPPER END, (KIPS OR TONNEF),
    (NEGATIVE=NET BUOYANCY OF A BUOY) =  2.00E -1    0.00E -1    0.00E -1

                 PARAMETERS OF SURFACE BUOY

THE BUOY IS LOCATED ON TOP OF SEGMENT NUMBER       2
BUOY WEIGHT IN AIR (LBS OR KG)        =  3.00E  3
DRAFT WHEN FREELY FLOATING (FT OR M)  =  1.50E  0
BUOY DEPTH (FEET OR METERS)           =  6.00E  0
POUNDS/FOOT OR KG/M OF IMMERSION      =  1.50E  3
DRAFT OF PADEYE WHEN BUOY IS FREE
  (FEET OR METERS BELOW WATERLINE)    =  8.00E  0

                 ENVIRONMENTAL INPUT

WATER DEPTH AT ANCHOR (FEET OR METERS)=  1.00E  3
VERTICAL DISTANCE FROM ANCHOR TO LINE
   TOP (FEET OR METERS)               =  1.01E  3
SEA BOTTOM SLOPE (DEGREES), POSITIVE
  WHEN SLOPING DOWN AWAY FROM BUOY    =  5.00E  0

                 NOMENCLATURE IN THE OUTPUT

SEG.        =SEGMENT NUMBER (SEGMENT 1 LATCHES ONTO THE ANCHOR)
SPAN        =HORIZONTAL SPAN OF THE STRETCHED LINE FROM ANCHOR TO FAIRLEAD,(FEET OR METERS)
SLOSB       =STRETCHED LENGTH OF LINE ON SEA BOTTOM (FEET OR METERS)
AVF         =ANCHOR VERTICAL FORCE (KIPS OR TONNEF)
HT          =HORIZONTAL TENSION (KIPS OR TONNEF)
DRAFT       =DRAFT OF SPRING BUOY (IF ANY) OR OF THE  BUOY IN THE TENSION-LEG CASE.
             THE DRAFT IS MEASURED FROM THE DRAFT OF THE FREE BUOY (FEET OR METERS)
T/B         =MAX TENSION/BREAKING STRENGTH RATIO
VTL         =VERTICAL TENSION AT THE LOWER END (KIPS OR TONNEF)
VTU         =VERTICAL TENSION AT THE UPPER END (KIPS OR TONNEF)
STULR       =STRETCHED TO UNSTRETCHED SEGMENT LENGTH RATIO
X AND Y     =HORIZONTAL AND VERTICAL SPANS OF THE STRETCHED SUSPENDED SEGMENT PORTION (FEET OT METERS)
             THE SPANS ARE MEASURED BETWEEN THE SEGMENT ENDS
LOSB        =UNSTRETCHED LENGTH OF LINE ON BOTTOM (FEET OR METERS)
S1          =UNSTRETCHED SUSPENDED LENGTH OF THE SEGMENT WHICH TOUCHES BOTTOM (FEET OR METERS)
FX AND FY   =CONVERGENCE FUNCTIONS
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Table 1 continued:
Results as functions of the horiz. line spans

SEG.	SPAN	SLOSB	AVF	HT	DRAFT	T/B	VTL	VTU	STULR	X	Y
1	3062.6	2946.8	.0	.9	77.16	.018	.0	5.4	.1000E 1	21.1	45.9
2						.028	5.6	8.6	.1028E 1	75.7	612.1
3						.011	1.8	3.8	.1000E 1	30.1	95.2
1	3145.0	2912.1	.3	3.0	73.58	.031	.3	9.1	.1000E 1	51.0	65.6
2						.042	9.3	12.3	.1035E 1	153.6	599.0
3						.024	5.5	7.5	.1000E 1	39.4	91.6
1	3227.5	2859.9	.6	6.7	69.57	.053	.6	14.6	.1000E 1	95.9	93.4
2						.063	14.8	17.8	.1044E 1	234.5	579.8
3						.044	11.1	13.1	.1000E 1	48.1	87.6
1	3309.9	2763.3	1.4	15.6	64.23	.098	1.4	25.0	.1000E 1	181.6	138.4
2						.108	25.2	28.2	.1058E 1	318.7	548.6
3						.085	21.5	23.5	.1000E 1	56.8	82.2
1	3392.4	2599.3	3.1	35.8	57.11	.187	3.1	43.2	.1000E 1	334.0	201.9
2						.195	43.4	46.4	.1079E 1	403.0	506.4
3						.166	39.7	41.7	.1000E 1	66.0	75.1
1	3474.8	2329.1	7.1	81.0	48.28	.366	7.1	74.2	.1000E 1	594.1	285.4
2						.373	74.4	77.4	.1110E 1	485.6	455.3
3						.330	70.6	72.6	.1000E 1	74.9	66.3
1	3557.2	1901.5	15.8	180.2	38.44	.732	15.8	125.7	.1000E 1	1014.8	388.0
2						.738	125.9	128.9	.1154E 1	565.6	399.8
3						.663	122.1	124.1	.1000E 1	82.6	56.4
1	3639.7	1269.7	33.7	385.6	28.83	1.459	33.7	206.8	.1000E 1	1643.7	505.0
2						1.464	207.0	210.0	.1218E 1	642.6	347.5
3						1.324	203.3	205.3	.1001E 1	88.4	46.8
1	3722.1	427.0	67.1	767.5	20.72	2.777	67.1	324.5	.1001E 1	2486.5	628.9
2						2.782	324.7	327.7	.1300E 1	717.9	305.1
3						2.523	320.9	322.9	.1001E 1	92.3	38.7

Table 1

```
PROGRAM SEML (SINGLE ELASTIC MOORING LINE)

MODE(TENSION-LEG OR CATENARY ?):     CATENARY

UNITS : METRIC(TONNEF,METERS,CENTIMETERS,KG),

                   INPUT AND NOMENCLATURE

   SEGMENT NUMBER (FROM ANCHOR)                1           2           3
UNSTRETCHED SEGMENT LENGTH (FT. OR M.)=     3.00E  3   6.00E  2   1.00E  2
UNIT WEIGHT IN WATER (LB/FT OR KG/M)  =     1.00E  2   5.00E  0   2.00E  1
FIRST CONSTANT OF STRETCH EQUATION    =     1.00E -9   1.80E -1   1.20E -9
SECOND CONSTANT OF STRETCH EQUATION   =     1.00E  0   5.00E -1   1.00E  0
BREAKING STRENGTH (KIPS OR TONNEF)    =     3.00E  2   3.00E  2   3.30E  2
WET WEIGHT ATTACHED TO THE SEGMENT
   UPPER END, (KIPS OR TONNEF),
   (NEGATIVE=NET BUOYANCY OF A BUOY)  =     2.00E -1   0.00E -1   0.00E -1

                   PARAMETERS OF SURFACE BUOY

THE BUOY IS LOCATED ON TOP OF SEGMENT NUMBER          2
BUOY WEIGHT IN AIR (LBS OR KG)        =     3.00E  3
DRAFT WHEN FREELY FLOATING (FT OR M)  =     1.50E  0
BUOY DEPTH (FEET OR METERS)           =     6.00E  0
POUNDS/FOOT OR KG/M OF IMMERSION      =     1.50E  3
DRAFT OF PADEYE WHEN BUOY IS FREE
  (FEET OR METERS BELOW WATERLINE)    =     8.00E  0

                   ENVIRONMENTAL INPUT

WATER DEPTH AT ANCHOR (FEET OR METERS)=     1.00E  3
VERTICAL DISTANCE FROM ANCHOR TO LINE
  TOP (FEET OR METERS)                =     1.01E  3
SEA BOTTOM SLOPE (DEGREES), POSITIVE
  WHEN SLOPING DOWN AWAY FROM BUOY    =     5.00E  0

                   NOMENCLATURE IN THE OUTPUT

SEG.       =SEGMENT NUMBER (SEGMENT 1 LATCHES ONTO THE ANCHOR)
SPAN       =HORIZONTAL SPAN OF THE STRETCHED LINE FROM ANCHOR TO FAIRLEAD,(FEET OR METERS)
SLOSB      =STRETCHED LENGTH OF LINE ON SEA BOTTOM (FEET OR METERS)
AVF        =ANCHOR VERTICAL FORCE (KIPS OR TONNEF)
HT         =HORIZONTAL TENSION (KIPS OR TONNEF)
DRAFT      =DRAFT OF SPRING BUOY (IF ANY) OR OF THE  BUOY IN THE TENSION-LEG CASE.
            THE DRAFT IS MEASURED FROM THE DRAFT OF THE FREE BUOY (FEET OR METERS)
T/B        =MAX TENSION/BREAKING STRENGTH RATIO
VTL        =VERTICAL TENSION AT THE LOWER END (KIPS OR TONNEF)
VTU        =VERTICAL TENSION AT THE UPPER END (KIPS OR TONNEF)
STULR      =STRETCHED TO UNSTRETCHED SEGMENT LENGTH RATIO
X AND Y    =HORIZONTAL AND VERTICAL SPANS OF THE STRETCHED SUSPENDED SEGMENT PORTION (FEET OT METERS)
            THE SPANS ARE MEASURED BETWEEN THE SEGMENT ENDS
LOSB       =UNSTRETCHED LENGTH OF LINE ON BOTTOM (FEET OR METERS)
S1         =UNSTRETCHED SUSPENDED LENGTH OF THE SEGMENT WHICH TOUCHES BOTTOM (FEET OR METERS)
FX AND FY  =CONVERGENCE FUNCTIONS
```

Table 1 continued:
Results as functions of the horiz. line spans

SEG.	SPAN	SLOSB	AVF	HT	DRAFT	T/B	VTL	VTU	STULR	X	Y
1	3062.6	2946.8	.0	.9	77.16	.018	.0	5.4	.1000E 1	21.1	45.9
2						.028	5.6	8.6	.1028E 1	75.7	612.1
3						.011	1.8	3.8	.1000E 1	30.1	95.2
1	3145.0	2912.1	.3	3.0	73.58	.031	.3	9.1	.1000E 1	51.0	65.6
2						.042	9.3	12.3	.1035E 1	153.6	599.0
3						.024	5.5	7.5	.1000E 1	39.4	91.6
1	3227.5	2859.9	.6	6.7	69.57	.053	.6	14.6	.1000E 1	95.9	93.4
2						.063	14.8	17.8	.1044E 1	234.5	579.8
3						.044	11.1	13.1	.1000E 1	48.1	87.6
1	3309.9	2763.3	1.4	15.6	64.23	.098	1.4	25.0	.1000E 1	181.6	138.4
2						.108	25.2	28.2	.1058E 1	318.7	548.6
3						.085	21.5	23.5	.1000E 1	56.8	82.2
1	3392.4	2599.3	3.1	35.8	57.11	.187	3.1	43.2	.1000E 1	334.0	201.9
2						.195	43.4	46.4	.1079E 1	403.0	506.4
3						.166	39.7	41.7	.1000E 1	66.0	75.1
1	3474.8	2329.1	7.1	81.0	48.28	.366	7.1	74.2	.1000E 1	594.1	285.4
2						.373	74.4	77.4	.1110E 1	485.6	455.3
3						.330	70.6	72.6	.1000E 1	74.9	66.3
1	3557.2	1901.5	15.8	180.2	38.44	.732	15.8	125.7	.1000E 1	1014.8	388.0
2						.738	125.9	128.9	.1154E 1	565.6	399.8
3						.663	122.1	124.1	.1000E 1	82.6	56.4
1	3639.7	1269.7	33.7	385.6	28.83	1.459	33.7	206.8	.1000E 1	1643.7	505.0
2						1.464	207.0	210.0	.1218E 1	642.6	347.5
3						1.324	203.3	205.3	.1001E 1	88.4	46.8
1	3722.1	427.0	67.1	767.5	20.72	2.777	67.1	324.5	.1001E 1	2486.5	628.9
2						2.782	324.7	327.7	.1300E 1	717.9	305.1
3						2.523	320.9	322.9	.1001E 1	92.3	38.7

Table 2

```
          PROGRAM SEML (SINGLE ELASTIC MOORING LINE)

          MODE(TENSION-LEG OR CATENARY ?):      TENSION-LEG

          UNITS : METRIC(TONNEF,METERS,CENTIMETERS,KG),

                     INPUT AND NOMENCLATURE

     SEGMENT NUMBER (FROM ANCHOR)                  1
UNSTRETCHED SEGMENT LENGTH (FT. OR M.)=     9.75E  1
UNIT WEIGHT IN WATER (LB/FT OR KG/M)  =     1.00E  2
FIRST CONSTANT OF STRETCH EQUATION    =     1.00E -9
SECOND CONSTANT OF STRETCH EQUATION   =     1.00E  0
BREAKING STRENGTH (KIPS OR TONNEF)    =     2.00E  2
WET WEIGHT ATTACHED TO THE SEGMENT
  UPPER END, (KIPS OR TONNEF),
    (NEGATIVE=NET BUOYANCY OF A BUOY) =     0.00E -1

                     PARAMETERS OF SURFACE BUOY

THE BUOY IS LOCATED ON TOP OF SEGMENT NUMBER        1
BUOY WEIGHT IN AIR (LBS OR KG)        =     5.00E  3
DRAFT WHEN FREELY FLOATING (FT OR M)  =     5.00E -1
BUOY DEPTH (FEET OR METERS)           =     6.00E  0
POUNDS/FOOT OR KG/M OF IMMERSION      =     1.00E  4
DRAFT OF PADEYE WHEN BUOY IS FREE
  (FEET OR METERS BELOW WATERLINE)    =     5.00E -1

                     ENVIRONMENTAL INPUT

WATER DEPTH AT ANCHOR (FEET OR METERS)=     1.00E  2
VERTICAL DISTANCE FROM ANCHOR TO LINE
  TOP (FEET OR METERS)                =     0.00E -1
SEA BOTTOM SLOPE (DEGREES), POSITIVE
  WHEN SLOPING DOWN AWAY FROM BUOY    =     0.00E -1

                     NOMENCLATURE IN THE OUTPUT

SEG.       =SEGMENT NUMBER (SEGMENT 1 LATCHES ONTO THE ANCHOR)
SPAN       =HORIZONTAL SPAN OF THE STRETCHED LINE FROM ANCHOR TO FAIRLEAD,(FEET OR METERS)
SLOSB      =STRETCHED LENGTH OF LINE ON SEA BOTTOM (FEET OR METERS)
AVF        =ANCHOR VERTICAL FORCE (KIPS OR TONNEF)
HT         =HORIZONTAL TENSION (KIPS OR TONNEF)
DRAFT      =DRAFT OF SPRING BUOY (IF ANY) OR OF THE  BUOY IN THE TENSION-LEG CASE.
            THE DRAFT IS MEASURED FROM THE DRAFT OF THE FREE BUOY (FEET OR METERS)
T/B        =MAX TENSION/BREAKING STRENGTH RATIO
VTL        =VERTICAL TENSION AT THE LOWER END (KIPS OR TONNEF)
VTU        =VERTICAL TENSION AT THE UPPER END (KIPS OR TONNEF)
STULR      =STRETCHED TO UNSTRETCHED SEGMENT LENGTH RATIO
X AND Y    =HORIZONTAL AND VERTICAL SPANS OF THE STRETCHED SUSPENDED SEGMENT PORTION (FEET OT METERS)
            THE SPANS ARE MEASURED BETWEEN THE SEGMENT ENDS
LOSB       =UNSTRETCHED LENGTH OF LINE ON BOTTOM (FEET OR METERS)
S1         =UNSTRETCHED SUSPENDED LENGTH OF THE SEGMENT WHICH TOUCHES BOTTOM (FEET OR METERS)
FX AND FY  =CONVERGENCE FUNCTIONS
```

Table 2 continued:
Results as functions of the horiz. line spans

SEG.	SPAN	SLOSB	AVF	HT	DRAFT	T/B	VTL	VTU	STULR	X	Y
1	.0	.0	10.2	.0	2.00	.099	10.2	20.0	.1000E 1	.0	97.5
1	1.3	.0	10.8	.2	2.05	.103	10.8	20.5	.1000E 1	1.3	97.4
1	2.6	.0	11.3	.5	2.10	.105	11.3	21.0	.1000E 1	2.6	97.4
1	3.9	.0	11.8	.7	2.16	.108	11.8	21.6	.1000E 1	3.9	97.3
1	5.2	.0	12.3	1.0	2.21	.111	12.3	22.1	.1000E 1	5.2	97.3
1	6.5	.0	12.9	1.2	2.26	.113	12.9	22.6	.1000E 1	6.5	97.2
1	7.8	.0	13.4	1.4	2.32	.116	13.4	23.2	.1000E 1	7.8	97.2
1	9.1	.0	14.8	1.9	2.46	.123	14.8	24.6	.1000E 1	9.1	97.0
1	10.4	.0	16.3	2.3	2.60	.131	16.3	26.0	.1000E 1	10.4	96.9
1	11.7	.0	17.7	2.8	2.75	.138	17.7	27.5	.1000E 1	11.7	96.8
1	12.9	.0	19.1	3.2	2.89	.145	19.1	28.9	.1000E 1	12.9	96.6
1	14.2	.0	20.9	3.8	3.07	.155	20.9	30.7	.1000E 1	14.2	96.4
1	15.5	.0	23.0	4.5	3.28	.165	23.0	32.8	.1000E 1	15.5	96.2
1	16.8	.0	25.1	5.2	3.48	.176	25.1	34.8	.1000E 1	16.8	96.0
1	18.1	.0	27.4	6.1	3.71	.188	27.4	37.1	.1000E 1	18.1	95.8
1	19.4	.0	29.9	7.1	3.97	.202	29.9	39.7	.1000E 1	19.4	95.5
1	20.7	.0	32.7	8.2	4.25	.216	32.7	42.5	.1000E 1	20.7	95.3
1	22.0	.0	35.5	9.3	4.53	.231	35.5	45.3	.1000E 1	22.0	95.0
1	23.3	.0	38.7	10.7	4.84	.248	38.7	48.4	.1000E 1	23.3	94.7
1	24.6	.0	41.9	12.1	5.16	.265	41.9	51.6	.1000E 1	24.6	94.3

Appendix

Derivation of elementary catenary equations

Consider an elastic segment shown in Figure 3. Let its unit weight be $w > 0$, and let the length between the ends and the lowest point be s_l and s_r, as sketched. The forces acting at the segment ends are the horizontal tension H, constant throughout the segment, and the vertical tensions L and U, as shown.

The balance of forces for the length s_r is

$$T_r \sin\theta = ws_r$$
$$T_r \cos\theta = H \tag{35}$$

Let

$$a = H/w \tag{36}$$

Then (35) can be rewritten as

$$\sin\theta = ws_r/T_r = ws_r/\sqrt{H^2 + (ws_r)^2} = s_r\sqrt{s_r^2 + a^2}$$
$$\cos\theta = H/T_r = a/\sqrt{s_r^2 + a^2} \tag{37}$$

But

$$\sin\theta = \frac{dv}{ds} = s_r/\sqrt{s_r^2 + a^2}$$
$$\cos\theta = \frac{du}{ds} = a/\sqrt{s_r^2 + a^2} \tag{38}$$

If this procedure is repeated for the left portion of the segment and the equations for both ends are combined, the horizontal and vertical segment spans, x and y respectively, take the integral form

$$x = u_r - u_l = \int_o^{s_r} \frac{ads}{\sqrt{s^2 + a^2}} - \int_o^{s_l} \frac{ads}{\sqrt{s^2 + a^2}}$$
$$y = v_r - v_l = \left[a + \int_o^{s_r} \frac{sds}{\sqrt{s^2 + a^2}}\right] - \left[a + \int_o^{s_l} \frac{sds}{\sqrt{s^2 + a^2}}\right] \tag{39}$$

Upon integration, the segment spans become

$$x = a\, ln \frac{a + \sqrt{s_r^2 + a^2}}{a + \sqrt{s_l^2 + a^2}} = \frac{H}{w}\, ln \frac{s_r w + \sqrt{(s_r w)^2 + H^2}}{s_l w + \sqrt{(s_l w)^2 + H^2}}$$
$$y = \sqrt{s_r^2 + a^2} - \sqrt{s_l^2 + a^2} = \tag{40}$$
$$= \frac{1}{w}[\sqrt{(s_r w)^2 + H^2} - \sqrt{(s_l w)^2 + H^2}]$$

Equation (39) can be written in terms of the vertical forces in place of the quantities $s_l w$ and $s_r w$,

$$x = \frac{H}{w}\, ln \frac{U + \sqrt{U^2 + H^2}}{L + \sqrt{L^2 + H^2}}$$
$$y = \frac{1}{w}\,[\sqrt{U^2 + H^2} - \sqrt{L^2 + H^2}] \tag{41}$$

where

$L = s_l w$ = vertical tension acting on the left (or lower) end,

$U = s_r w$ = vertical tension acting on the right (or upper) end.

Equations (41) are remarkably general. They are valid for $w < 0$ and $w > 0$, as well as for $U \gtrless 0$ and $L \gtrless 0$.

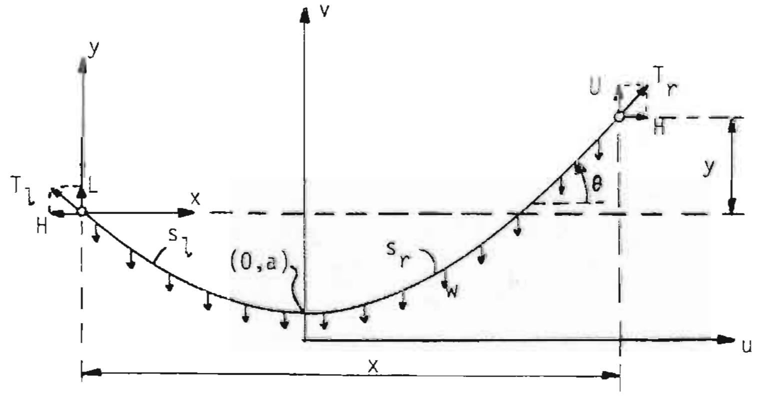

Figure 3. Catenary segment.

Nomenclature

a	catenary weight parameter used in derivations
A_i	i-th segment effective cross-sectional area
b	total unstretched length of line portion on sea bottom
b_i	unstretched length of the i-th segment portion on sea bottom
B	total stretched length of line portion on sea bottom
B_i	stretched length of the i-th segment portion on sea bottom
C_i, C_i^b	stretch factors for the suspended and on-the-bottom portions of the i-th segment
d	draft of the buoy/vessel in the freely-floating (unmoored) condition
D	water depth at anchor
E_i	Young's modulus of i-th segment
f_o	length of the line connecting the buoy bottom to the top of the K-th segment
f_y	merit function used in iterations
F_i	breaking or proof load of i-th segment
G_i	weight in water or net buoyancy of the element attached to the top of i-th segment
H	horizontal tension in mooring line
i	index of the line segments (1 = lowest)
K	segment number on top of which the spring buoy is attached
L_i	vertical tension at the lower end of i-th segment
N	number of segments in the line
p_i, q_i	constants in the stress-strain equation for i-th segment
Q_i	symbol used in derivations
R	vertical reaction at anchor
s_i, S_i	unstretched and stretched lengths of i-th segment
s_l, s_r	unstretched lengths of the left and right portions of a suspended segment
s_i	stretch of i-th segment
t	touch-down point of the mooring line
T	axial tension
T_l, T_r	axial tensions at the left and right ends of a segment
u, v	coordinate system for a segment
u_r, v_r	horizontal and vertical spans of the right-hand side portion of a segment
u_l, v_l	as u_r and v_r but for the left portion
u_b	draft of the spring buoy/vessel in excess of the draft when freely-floating, d
U_i	vertical tension in the upper end of i-th segment
$V_i(s)$	vertical tension in i-th segment, as a function of the position along the segment, s
X, Y	total horizontal and vertical spans of a mooring line
x_i, y_i	horizontal and vertical spans of the unstretched i-th segment
X_i, Y_i	horizontal and vertical spans of the stretched i-th segment
$Z(u_b)$	net buoyancy of the spring buoy/vessel as a function of its net draft.
α	slope of the sea bottom at the anchor