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Fractal Holography: a geometric re-interpretation of cosmological large scale structure

Jonas R. Mureika
Loyola Marymount University, jmureika@lmu.edu

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J. R. Mureika
Department of Physics, Loyola Marymount University, Los Angeles, CA 90045-8227
Email: jmureika@lmu.edu

Abstract The fractal dimension of large-scale galaxy clustering has been reported to be roughly $D_F \sim 2$ from a wide range of redshift surveys. If correct, this statistic is of interest for two main reasons: fractal scaling is an implicit representation of information content, and also the value itself is a geometric signature of area. It is proposed that a fractal distribution of galaxies may thus be interpreted as a signature of holography ("fractal holography"), providing more support for current theories of holographic cosmologies. Implications for entropy bounds are addressed. In particular, because of spatial scale invariance in the matter distribution, it is shown that violations of the spherical entropy bound can be removed. This holographic condition instead becomes a rigid constraint on the nature of the matter density and distribution in the universe. Inclusion of a dark matter distribution is also discussed, based on theoretical considerations of possible universal ΛCDM density profiles.

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1 Introduction

The popular notions of fractals revolve around spatial power law scaling, physical self-similarity, and structural recursiveness [1]. Mathematically, this relationship assumes the general form

$$N(r) \sim r^{D_F} \quad . \tag{1}$$

where D_F is the fractal dimension and r is the scale measure. The quantity $N(r)$ represents the characteristic of the distribution that exhibits the fractal behavior. Measurement of fractal statistics for a wide range of physical phenomena has been addressed over the years, ranging from the shape of coastlines to the structure of clouds, and pertinent to this paper, the large scale distribution of visible matter in the universe [2].

It should be emphasized that the meaning of the fractal dimension is not only statistical, but it also has geometric significance. Topological considerations constrain the fractal dimension of a distribution to be less than (or equal) to that of the space in which the structure is embedded [3]. Furthermore, when a fractal dimension coincides with an integer dimension, it is possible to make the association between the structure under consideration and the geometry associated with the dimension. That is, a distribution with fractal dimension of

Survey	D_F	Approx. Size
CfA1	1.7 (0.2)	1800
CfA2	~ 2	11000
SSRS1	2.0 (0.1)	1700
SSRS2	~ 2	3600
LEDA	2.1 (0.2)	75000
IRAS 1.2/2 Jy	2.2 (0.2)	5000
Perseus-Pisces	~ 2.1	3300
ESP	1.8 (0.2)	3600
Las Campañas (LCRS)	2.2 (0.2)	25000
SDSS (r1)	~ 2	$2 \times 10^5 - 1.5 \times 10^6$

Table 1: Galaxy fractal dimension calculations for various redshift surveys (compiled from [5],[6],[7],[10]).

$D_F = 0$ is described as a point distribution, $D_F = 1$ a linear distribution, $D_F = 2$ a surface distribution, and $D_F = 3$ a volumetric or space-filling distribution.

A hierarchically-structured universe is a recurrent theme in our understanding of Nature. Since at least the early sky maps of Charlier [4], it has been suggested that galaxies do *not* cluster in a random fashion, but rather appear in clumps interspersed by voids. The advent of deep sky redshift surveys brought with it a surge interest surrounding the exact nature of large-scale galaxy distributions in the observable universe. An overwhelming number of independent estimates of the galaxy clustering fractal dimension, obtained from a variety of sources seem to unanimously suggest that this statistic has a value of or around $D_F = 2$. Up to the release of the SDSS data, the various redshift surveys had probed depths up to at least $10h^{-1}$ Mpc and confirmed the fractal scaling behavior. Extrapolating the analysis to include superclustering structure suggested this behavior continued well up to $100 - 1000\ h^{-1}$ Mpc [5], with no apparent transition to homogeneity.

The newest SDSS redshift data confirms the $D_F \sim 2$ to a high precision up to $20h^{-1}$ Mpc, but with the correlation weakening to homogeneity at distances of $70h^{-1}$ Mpc. Alternate analyses suggest that the transition to homogeneity is not observed, but instead the fractal scaling continues up to $200h^{-1}$ Mpc [6]. The authors of Reference [7] perform a two-dimension multifractal analysis on SDSS projected data, deducing that the appropriate scaling is confirmed to dimensions $D_q \in (1.7, 2.2)$ for all positive and negative q.

On the whole, this observational data suggests a (local) violation of the cosmological principle. Geometrically speaking, a homogeneous visible universe should manifest itself as an $N(r) \sim r^3$ distribution. In terms of the fractal dimension, this is a volumetric scaling with a dimension $D_F = 3$, synonymous with the lack of any preferential direction. The origins of the observed large scale structure in the universe are unknown, although it is commonly believed that it has arisen from anisotropically-distributed quantum fluctuations in the pre-inflation epoch. The recent analysis of the SDSS data [8] coupled with emerging CMB data from WMAP [9] data has helped to isolate probable cosmological parameters to determine viability of formation models.

Although the origin of this structure is still a mystery, the most successful models are

ΛCDM gravitational collapse scenarios in the early universe [11]. The basic framework of such cluster-formation models relies on perturbations in the local background curvature due to quantum fluctuations during the inflation epoch. Such models successfully reproduce a wide range of observable clustering features, including power spectra and correlation functions, as well in many cases the cited fractal dimensions for galaxy clustering [12, 13, 14, 15]. More recently, such models have been extended to galaxy and quasar clustering [16], with the added advantage of introducing universal dark energy constraints. In this sense, the ΛCDM scenarios are the model logical and consistent explanation for the present inhomogeneous matter distributions. In fact, very recent observational data [17] supports the filamentary dark matter "scaffolding" that are a consequence of the models.

Despite the fact that the apparent success of the ΛCDM models seems to put to rest any mysteries surrounding the origin of large scale structure formation, this paper will focus on a much different mechanism. which can be explained by adopting a new perspective on how matter and gravitation are allowed to behave? Fractal behavior is generally not associated with equilibrium growth, and thus most models of large scale structure evolution do not predict its existence. As the aforementioned evidence suggests, however there appears to be a uniformly-defined fractal distribution of matter in the universe, at the very least up to some as-yet unkown scale length. The use of entropy to represent fractal structure stems from the implicit relation between entropy and information (this is discussed in the concluding section of this paper). Fractal – and moreover multifractal – statistics quantify the nature in which information is encoded or distributed in a system. It is the intention of this paper to highlight this connection between information, entropy, and fractality.

Before proceeding, it would be negligent to ignore the controversy surrounding the fractal interpretation of large scale structure, which is by no means an established fact. Dubbed the "fractal debate", there has been an ongoing discussion of whether or not the analyses indicating fractal clustering have been performed with the proper data treatment, most notably a reliable estimate of the redshift distance from otherwise two-dimensional projections. There are clearly two opposing opinions on the exact nature of any characteristic scale lengths and clustering that might exist in clustering. The interested reader is directed to some comprehensive and competing summaries of both sides of the debate in such references as [5, 18, 19, 20, 21]. The availability of more redshift data will either confirm, deny, or further muddle this issue. For the purposes of this discussion, however, the fractal model will be assumed to be correct.

More pertinent to this paper, it should be noted that fractals themselves are members of a fundamental class of statistical object whose basis lies in the heart of information theory. A q-multifractal is defined by the measure partition $Z(q,r) = \sum_i [p_i(r)]^q$, and dimensions $D_q = \left(\frac{1}{q-1}\right) \frac{d\log[Z(q,r)]}{dlog[r]}$. where q is any integer and p_i is the local spatial density of the fractal object within a sphere of radius r [3]. The traditional fractal dimension is obtained in the limit $q = 0$, but when $q \to 1$ this quantity becomes $I(r) = -\sum_i p_i(r)\ \log p_i(r)$, $D_1 = \frac{d\log I(r)}{d\log r}$. The function $I(r)$ is the Shannon information entropy [22]. In the case of a monofractal distribution, all dimensions D_q collapse to the single value D_0, which is the fractal dimension. In this respect, the fractal dimension may be seen as a representation of a system's entropy measure and content.

2 Information theory and the holographic principle

Information content and entropy have entered the area of the long-standing dichotomy between classical and quantum gravity. In early works by Beckenstein [23] it was suggested that the maximum entropy contained within a black hole was determined not by its volume, but rather the horizon area $\mathcal{A}_H$. This limit, known as the Beckenstein bound, placed stringent constraints on how information could distribute itself within a region of space. It was further generalized as the spherical entropy bound,

$$S(V) \leq \frac{A}{4} \tag{2}$$

where $S(V)$ is the entropy contained in a volume of space V, and A is the area of the (spacelike) boundary of V (in units of the Planck area).

Bousso has shown that each of these entropy bounds can be understood as classes of a more general theory known as the holographic principle (HP) [24]. An unproven hypothesis, the HP suggests that there exists a deeper geometric origin for the total number of possible quantum states which can occupy a spatial region. In its most general formulation, the HP states that $S(B) \leq \partial B/4$, where B is some region, ∂B its boundary, and $S(B)$ the entropy contained in B. Although most instances are subject to specific failures, the most radical formulation of the HP – Bousso's Covariant Entropy Conjecture – proposes that the entropy bounded by a region B defined by the *light sheets* of a backward-pointing null cone obeys a holographic-type relationship. The various entropy bounds which form the holographic principle place rigid constraints on the number of possible entropy states which can occupy a region of space [24].

The HP is novel in its motivations: the physics of a spatial n-dimensional region are defined by dynamical systems which exist exclusive on the region's (n-1)-dimensional boundary. The most promising support of this theory is the AdS/CFT correspondence [25], which explicitly connects via a one-to-one correspondence the framework of a 5D string theory in anti-deSitter space with a conformal quantum field theory on the 4D boundary.

3 Cosmology and holography

Fischler *et al.* [26, 27, 28, 29] have proposed an extensive cosmological version of the theory, primarily based in part on the spherical entropy bound. Dubbed "Holographic Cosmology", the framework presents an alternate inflationary evolutionary model for large scale structure in which structure evolves from evaporating primordial quantum black holes. This model not only matches current observation but also explains the flatness and horizon problems. A similar proposal based on the original work of Susskind and Fischler is discussed in [30], which proposes a "cosmic holography" bound in FRW universes of positive, flat, and negative curvature. Further comparisons and contrasts between holography and cosmology are offered in [31], in which constraints from inflation are the focus. Similarly, the authors of [32] also show that power spectrum correlations and suppression in the CMB may by holographic in origin. References [33] discuss holographic implications for (2+1)-dimensional cosmological models.

As derived in [26], assuming a homogeneous and isotropic universe with constant mean density, it is possible to define a (co-moving) entropy density σ such that the total entropy with a volume V is [24]

$$S = \int_V d^3x\sqrt{h}\sigma \tag{3}$$

which can be written $S = \sigma V$, where σ is the (volumetric) entropy density. The entropy condition is thus

$$\sigma V \leq \frac{A(V)}{4} \quad , \tag{4}$$

where $A(V) = 4\pi r^2$ is the bounding area of the volume V (in flat space). Including the r-dependence, the entropy bound is

$$r^3\sigma \leq \frac{3r^2}{4} \quad , \tag{5}$$

and so it can easily be shown [24] that the spacelike entropy bound is violated for sufficiently large values of $r > 3/4\sigma$.

The aforementioned models rely on quantum mechanical entities (black holes) to describe the basic units of the entropy bound, while proposals like the one presented herein assume that galaxies are the key. The application of holographic bounds makes the assumption that these objects are themselves in some way a fundamental "unit" of entropy. It is, however, reasonable to make this *a priori* postulate.

It has long been theoreized that galaxies themselves are the result of gravitational clustering around supermassive black holes that have either grown from accretion, or from combining with smaller primordial black holes (see the various references [34, 35] and references therein). Although supermassive black holes had been thought only to populate the bulges of active galactic nuclei, growing evidence suggests that such objects may also be found in type I Seyfert galaxies [36] and high-redshift blazars [37]. If one considers black holes to be the "seeds" of every galaxy, then these objects may be understood to represent a universal evolution toward a maximal spatial entropy state, independent of the type or size of galaxy.

4 A fractal connection to holography?

The work presented herein is similar in inspiration to that of Fischler *et al.*, and like those referenced works promotes the notion that holography should be a viable candidate for a constraint of structure evolution. Holographic constraints will first be applied to visible matter, but a discussion including dark matter distributions will follow in Section 5. Since it is somewhat different from the previous cosmological holographies proposed in the literature, it might be appropriate to label this version as "fractal holography".

It is the different spatial dependences of area and volume that allows the inequality (4) to be violated. With respect to Equation 1, however, one can reformulate the "problem" of large scale fractal clustering by focusing not on the apparent break from homogeneity and isotropic scaling, but rather by highlighting the specific *geometry* of the scaling. Since the redshift surveys cited in Table 1 indicate that large-scale matter is distributed according to a $D_F = 2$ scaling, it seems more appropriate to describe the entropic content by the mean "surface" entropy density ξ. That is, each object contributes an average entropy $S(V)/N$ and there are $N \sim r^{D_F}$ objects. It should be noted that the fractal power laws are derived from average density considerations, so such an argument is certainly well-founded. Fractal large scale structure thus states that within a sphere of radius r, the number of cosmological objects is a function of *area* (r^2).

Thus, within a spherical volume of radius r, the number of galaxies $N(r)$ must be *proportional* to the surface area of the region's boundary,

$$N(r) \propto \partial V(r) = A(r) \ , \tag{6}$$

so that the entropy contained with a region V is

$$S(V) = \alpha \xi A \ \ , \tag{7}$$

where $\alpha > 0$ is the proportionality constant. The above relation suggests that the distribution of matter in the universe has perhaps a more fundamental and geometric origin.

In this case, a holographic-type space-like entropy bound is precisely given as

$$S(V) = \bar{\xi} A \leq \frac{A}{4} \ \ , \tag{8}$$

where for simplicity the proportionality constant has been absorbed into the surface density term, $\bar{\xi} = \alpha\xi$. Due to the $A \sim r^2$ dependence of each component of this inequality, the spatial dependence vanishes and what is left is a truly scale-invariant bound. Specifically, the violation of the space-like entropy bound is eliminated, and instead is replaced by rigid constraints on the surface entropy density, and hence the *geometry* of the matter distribution:

$$\bar{\xi} \leq \frac{1}{4} \ . \tag{9}$$

What might be the value of ξ? The fractal distribution of galaxies extends to about 10 Mpc, or 10^{58} Planck length units. The area of the bounding sphere is thus on the order of 10^{116} area units. The entropy content of the *entire* visible universe is on the order of 10^{90} [31], so even if a sizable fraction is represented in this fractal distribution, this implies the "surface" density is no greater than $\xi \sim 10^{-24}$ or so. The value of the proportionality constant α thus is the key to the inequality. Unless α is of exceedingly high order of magnitude, though, it is unlikely that this bound will ever be violated.

4.1 Entropy bounds for fractal dimensions near 2

Although the observational evidence points to a fractal scaling dimensions of $D_F = 2$, this exact geometric signature could be a coincidence. If such is the case, then the spherical entropy bound (8) will not be scale invariant. However, implications of the bound become even more interesting if one follows the prescription for non-integer scaling dimensions around $D_F = 2$.

Following this philosophy, (8) can be expressed as

$$\begin{aligned} \chi r^{D_F} &\leq \frac{A}{4} \\ \frac{\chi}{\pi} &\leq r^{2-D_F} \end{aligned} \tag{10}$$

where χ is the "fractal number density" of the distribution. So, violations of the entropy bound will occur whenever (up to geometric factors)

$$r > \left(\frac{1}{\chi}\right)^{1/(D_F-2)} \quad . \tag{11}$$

If the fractal dimension is slightly higher than 2, the bound will ultimately be violated for a large enough sphere. However, the relative radius of the sphere will be much greater than in the case of homogeneity. In the case $D_F = 3$, this reduces to the violation derived in [26].

4.2 Transitions to homogeneity

Current observation suggests that the fractal distribution of matter may transition to homogeneity at large distances. In this case, the entropy density (7) and scale-invariant entropy bound (8) are no longer applicable, at least on a global scale.

In addition to the references discussed in Section 1, a recent analysis [38] has determined that number of luminous red galaxies (LRGs) shows a well-defined $D_F = 2$ fractal behavior up to scale lengths of at least 20 h^{-1} Mpc, and a smooth transition to homogeneity ($D_F = 3$) beyond scales of 70 h^{-1} Mpc. Reference [39] confirms $D_F \sim 2$ fractal clustering behavior to a scale of 40 Mpc, using independent analysis techniques such as the nearest neighbor probability density, the conditional density, and the reduced two-point correlation function.

In terms of the holographic model, in the simplest of cases, consider a spherical region of radius R (in flat space) in which the distribution of matter is fractal with $D_F = 2$. Within this region, the entropy constrain obeys that described in Equation 7, *i.e.* $S_{\mathrm{F}}(r) \sim \bar{\xi} r_F^D$. For separation scales $r > R$, the distribution resembles a homogeneous one, and the total entropy is now described by a volumetric distribution with density σ. At the transition scale $r = R$, these descriptions of entropy must agree. That is, the total entropy within a sphere of radius $r = R$ should be $S_{\mathrm{F}}(R) = S_{\mathrm{H}}(R)$, where $S_{\mathrm{F}}(R) = 4\pi\bar{\xi}R^2$ and $S_{\mathrm{H}} = 4/3\pi\sigma R^3$. This implies that $R = \frac{3\bar{\xi}}{\sigma}$.

Potential violations of the holographic principle are now re-introduced at scales $r > R$, as described by Equation 5. However, the requirement of statistical continuity in the description of the entropy also requires that $S'_{\mathrm{F}}(R) = S'_{\mathrm{H}}(R)$. This provides an additional constraint equation on the value of R, in this case $R = \frac{2\bar{\xi}}{\sigma}$, assuming the only radial dependence in the holographic bound is in the geometric term (area and volume).

Hence, fractal holography implicitly supports the well-known observation of a slow transition to homogeneity over distances of several megaparsecs. A similar holographic model could represent the transition entropy in the most general form $S_{\mathrm{Trans}}(r) \sim \rho(r) r^{D(r)}$, where both the entropy "density" ($\rho(r)$) and the fractal dimension itself ($D(r)$) become functions of the scale length. This can be cast as a boundary-constrained problem, with the requirements $S_{\mathrm{F}}(R_1) = S_{\mathrm{Trans}}(R_1)$, $S_{\mathrm{H}}(R_2) = S_{\mathrm{Trans}}(R_2)$, $S'_{\mathrm{F}}(R_1) = S'_{\mathrm{Trans}}(R_1)$, $S'_{\mathrm{H}}(R_2) = S'_{\mathrm{Trans}}(R_2)$. Appropriate constraints on the values of $D(r)$ and its derivative, for example, could be isolated from correlation and fractal analyses.

It should also be noted that a critical discussion of the meaning of "transition to homogeneity" found in [40] indicates that there could be two possible interpretations of what it means to transition to homogeneity. This could be in terms of a trivial correlation function at $r = R$ (the usual transition boundary), as well as a more long-range scale λ that could be greater than the horizon distance. Regions measured at scales $R < r < \lambda$ are not strictly homogeneous, but rather are analogous to a fluid at the critical point.

An oft-cited argument for the necessity of a transition to homogeneity stems from the exceedingly homogeneous distribution of anisotropies in the cosmic microwave background (CMB). It is important to note, however, that inferring homogeneity of a three-dimensional distribution from its two-dimensional projection is not trivial. From the point of view of fractal statistics, for example, a homogeneous surface distribution would coincide with a fractal dimension of $D_S = 2$. The fractal projection theorem [41] states that for any fractal with dimension D_F, its projection onto a sub-plane of dimension $D_P \leq D_F$ will itself have a dimension of D_P. That is, the dimensionality of the projective distribution "saturates" the sub-plane. It is thus possible that the perceived homogeneity of the CMB may not truly correlate to the homogeneity of volumetric spatial distributions at the time of last scattering. In this case, the original distribution of anisotropies may well have obeyed a holographic-type "area" constraint, like that discussed herein.

4.3 Scale evolution and fractal holography

There is ongoing debate as to whether or not the fractal distribution of visible matter extends indefinitely to beyond 1000 h^{-1} Mpc. If in fact the entire universe is governed by the $D_F \sim 2$ fractal distribution, the fractal holographic condition can place a bound on its expansion rate.

For a sphere whose radius R_H is the horizon distance, the usual holographic bound in d spatial dimensions is [26]

$$\sigma R_H(t)^d < [a(t)R_H(t)]^{d-1} \quad . \tag{12}$$

with $a(t) \sim t^p$ the scale factor of the universe, p an expansion parameter, and $R_H(t) = \int_0^t a(t')dt \sim t^{1-p}$. The left hand term in the inequality assumes that the entropy scales volumetrically. Since σ is small, it has been demonstrated that the inequality is satisfied throughout the history of the universe if $p > 1/d$.

Adopting the fractal interpretation and setting $d = 3$, this becomes

$$\bar{\xi} R_H(t)^2 < [a(t)R_H(t)]^2 \quad . \tag{13}$$

The new constraint on the evolution parameter is thus $\bar{\xi} < t^{2p}$, which is almost certainly always satisfied for an arbitrary choice of p.

5 Inclusion of dark matter

So far, the discussion of fractal holography has excluded dark matter. Clearly, any viable model for large scale structure must replicate more than the visible matter distributions, but also the "invisible" ones. Although the spatial structure of halo dark matter density profiles can easily be inferred from galaxy rotation curves, it is uncertain exactly what the large scale distribution looks like.

Since dark matter is believed to make up well over 90% of the material content of the universe, no cosmological model would be complete without paying due attention to this mystery. Unfortunately, not much is known about the actual form of the distribution of dark matter in the universe. The best models we have for density distributions are those of “small scale” dark matter halo structures derived from galaxy rotation curves, such as the NFW profile [42], which suggest density profiles of the form $\rho_{\rm DM} \sim r^{-2}$. While these reflect the distribution of halo dark matter, they unfortunately offer no insight into the larger scale structure.

Based on a simple inverse-square density profile for dark matter, the authors of Reference [43] have shown that a fractal distribution of galaxies is not inconsistent with a homogeneous distribution of all matter, by virtue of the fact that the dark matter density profile is the functional reciprocal of the galaxy number count. The authors further note that this implies a different fractal correlations for luminous matter ($D_F = 2$) and dark matter ($D_F = 3$).

Some numerical simulations of ΛCDM cosmologies suggest that dark matter halo profiles should roughly echo that of the matter distribution in the universe [44]. In particular, a universal density profile derived from N-body simulations has shown that dark matter may cluster in a hierarchical fashion [45].

It has more recently been suggested that all baryonic matter can be distributed in an r^2 fractal manner, by appealing to alternative theories such as Modified Field Theory [46]. Such a description is consistent with both the Cosmological Principle, as well as the Silk Effect [47], and can produce a gravitationally-stable fractal clustering (the interested reader is referred to [46] for further details).

For simplicity, assume the density profile of dark matter is hierarchical, according to the power law $\rho_{\rm DM} \sim r^{-\gamma}$. This implies that the number of objects within a region of radius r is $N_{\rm DM} \sim r^{3-\gamma}$, which we may associated with a fractal dimension $D_{\rm DM} = 3 - \gamma$. The value of γ ranges depending on the literature source, from $\gamma = 1.5$ [48] to $\gamma = 2.1 - 2.5$, thus the corresponding fractal dimensions would range between $D_{\rm DM} \sim 1.5 - 2.5$. In this case, if $D_{\rm DM} \leq 2$, the holographic constraint behaves as with luminous matter (and thus is potentially not violated).

The most promising glimpse at possible larger-scale distributions of dark matter has been reported by the COSMOS collaboration [17]. These results suggest a largely filamentary clustering of dark matter in “rods”, with the more familiar halo clusters forming at “vertices” of several rods. The extact spatial extent of the observed filaments is difficult to extract from the present data, and specific clustering measurements are currently underway[1]. Preliminary results suggest that the dark matter filaments can extend up to 30 Mpc or more [17]. Nevertheless, the global geometry of such inhomogeneous filamentary structures would be consistently with a linear dark matter scaling (similar to a fractal scaling of $N(r) \sim r$), which would not violate any holographic constraints.

5.1 General density distributions and holographic charge

As a final comment on the geometric re-interpretation of the holographic principle, consider the general case of matter distributions with fractal dimensions $D_F = 0, 1, 2,$and 3. The holographic constraints are thus dimensionally written (omitting some constants)

[1] R. Massey, personal communication

$$
\begin{aligned}
S_0 \ , \ S_1 \ , \ S_2 \ , \ S_3 \ &\leq \ \frac{A}{4} \\
\delta \ , \ \lambda r \ , \ \xi r^2 \ , \ \sigma r^3 \ &\sim \ r^2 \\
\frac{\delta}{r^2} \ , \ \frac{\lambda}{r} \ , \ \xi \ , \ \sigma r \ &\sim \ 1
\end{aligned} \tag{14}
$$

The above expressions, cast in this manner, now become geometrically reminiscent of field strengths for various charge distributions. The geometry in question corresponds to the dimensionality of the fractal: point, linear, surface, and interior field of a continuous distribution. The functional similarity to Gauss' Law comes from the area term of the inequality, and in this sense one might interpret Equation 14 as some variety of "holographic field" equation (although the similarities are most likely only superficial).

6 Conclusions and future directions

The inspiration for cosmological holography rests in the notion that quantum scale entropic physics can be arbitrarily expanded to any stable gravitational system. It is thus possible that such holographic constraints in the early universe led to the formation and non-homogeneous distribution of anisotropies. Furthermore, the previous discussion suggests that the link between holographic area bounds and fractal $D_F = 2$ scaling may be related. The galaxy number counts can be directly related to the entropy content by cosmological considerations like those discussed herein.

How is such a theory useful to further understanding cosmology and the origins of large scale structure? At the very least, it helps to place somewhat rigid constraints on the nature of the clustering, as discussed in Section 4.2 – that is, where the clustering can be represented as "fractal", where it might be homogeneous, and over how far a distance the transition can occur. Hence, a cosmological model can be built using the holographic constraints placed on the density distributions and their associated spatial extents. In this sense, such a theory is not limited to only the "fractal" region of galaxy distributions, but can extend indefinitely and still provide useful boundary constraints.

Nothing in this proposal changes the fundamental origins of clustering. Galaxies can still form as standard theory dictates. Per the discussion in Section 3, galaxies do not replace anything as the fundamental unit of structure, since they themselves are likely "perturbations" to optimal clustering around black holes. Since holographic theories have evolved from the study of such objects, it is not out of the question to expect galaxies to abide by some similar form of such principles.

Granted, the success or failure of any holography-inspired theory hinges on the success of the holographic principle itself. At present, this is still a largely-hypothetical and unproven conjecture with limited (but growing) theoretical support. Since the holographic bounds are derived as a fundamental "entropy saturation limit" of the universe, violations thereof would likely imply that the base assumption is incorrect.

This paper has not considered the holographic bounds introduced in open and closed universes, but there are several reasons for this omission. First, the wealth of observational data suggests that the universe is most likely flat. Secondly, while it is still possible to measure fractal dimensions in curved spaces using geodesic radii (via methods such as correlation analyses), the elegant geometric interpretation of the fractal dimension (Equation 1) is not as easily realized.

Thus the connection between information theory, gravitation, and geometry is a common "theme" for fractal large scale structure. At the very least, the observed fractal distribution behavior of galaxies could be understood to be a large scale bookend principle to holography. Redshift survey results provide strong evidence that the number counts scale as an area, but in order to verify a deeper connection future analyses should also focus on the pre-factor of the fractal relationship. Fractal clustering of large-scale structure may well represent either a manifestation of holographic entropy bounds, or the end result of a cosmological holography model, and future studies should adopt such a re-interpretation to explore new implications of the data.

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