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Vector unparticle enhanced black holes: exact solutions and thermodynamics

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Abstract

Tensor and scalar unparticle couplings to matter have been shown to enhance gravitational interactions and provide corrections to the Schwarzschild metric and associated black hole structure. We derive an exact solution to the Einstein equations for vector unparticles, and conclusively demonstrate that these induce Riessner-Nordström (RN)-like solutions where the role of the "charge" is defined by a composite of unparticle phase space parameters. These black holes admit double-horizon structure, although unlike the RN metric these solutions have a minimum inner horizon value. In the extremal limit, the Hawking temperature is shown to vanish. As with the scalar/tensor case, the (outer) horizon is shown via entropy considerations to behave like a fractal surface of spectral dimension $d_H = 2d_U$.

1 Introduction

Recently, it was proposed that there could be a conformally scale-invariant particle sector of unknown composition with a non-trivial IR fixed point [1,2], at which stronger couplings to the standard model emerge. Dubbed "unparticle physics" because of the non-intuitive phase space structure, its introduction has caused a flurry of research into modifications to known physics and post-TeV predictions. Accelerator phenomenology has been the main emphasis in the literature [3–10] [11,12], but astrophysical/cosmological [13–18] [19–24] [25–27], low to ultra-high neutrino phenomenology [28–35] and general quantum field theory [36–41] [42,43] applications have been of extreme importance as well.

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Constraints on the unparticle parameters Λ_U, the BZ messenger mass M_U, and the unparticle and Banks-Zaks dimensions d_U, d_{BZ} are obtained through limits on measurable accelerator phenomenology, astrophysical and cosmological observations. The aforementioned parameters serve to fix the energy and distance scale at which the interactions become relevant. It has been shown that, if $\Lambda_U \sim 1$ TeV, then strict limits may be placed on the messenger M_U for various values of the unparticle dimension d_U. Higher values of d_U imply lower values of M_U, whose value could be as small as a few hundred TeV.

One of the most intriguing aspects of unparticle physics is that interactions with standard model particles effectively modify the usual gravitational coupling strength [26,27,45]. Dubbed "ungravity", in the Newtonian limit this is most likely to be observed as deviations in planetary orbits and perihelion precession [25,26] on large scales, as well as constrain Big Bang Nucleosynthesis [14], dark energy [24] and even entropic gravity [42]. Conversely, the very small-scale behavior of scalar and tensor ungravity begins to mimic that of n large extra compactified dimensions [44], but with $n \to 2d_U - 2$ and the Newtonian potential proportional to $1/r^{2d_U-1}$ [45–47]. The interesting property here is that, since d_U can be non-integer, ungravity reproduces the phenomenology not only of standard extra-dimensional physics, but also of a "fractal" spacetime.

It was conjectured by perturbative arguments that such a modification to the Newtonian potential would result in unparticle-driven mini-black hole creation in high energy collisions [45,46]. More recently, exact solutions to Einstein's equations were derived for unparticle interactions with matter, showing that the previous approximation holds in both the weak and strong gravity limits [48].

This paper will address the influence of vector unparticle interactions with matter, and the respective solutions of the Einstein equations. We show that, as in the scalar/tensor case, vector unparticles modify the metric in an analogous fashion and admit black hole solutions enhanced by the unparticle parameters. Since vector ungravity is repulsive, however, the resulting horizon and singularity structure is comparable to Riessner-Nordström class of metrics, where the "charge" is a composite of unparticle parameters. We also discuss the unique thermodynamics of such black holes, and consider the associated implications for the spacetime dimensionality..

2 Basics of unparticle physics

Unparticle physics is characterized by its non-integer scaling dimension d_U in phase space, making it "look like" a system of d_U fundamental particles. A weakly-coupled Banks-Zaks (BZ) field [49] exchanges a massive particle M_U with standard model field, suppressed by non-renormalizable interactions

$$\mathcal{L} = \frac{1}{M_U^{d_{SM}+d_{BZ}-4}} O_{SM} O_{BZ} \quad . \tag{1}$$

Here, O is the unparticle operator, which may possess any Lorentz type (scalar, vector, tensor, spinor). The dimensions d_{SM} and d_{BZ} correspond to the standard model and Banks-Zaks fields.

The coupling M_U will run below some energy scale $\Lambda_U < M_U$, and the field transmutes to the unparticle operator O_U of dimension $d_U \neq \mathrm{d}_{BZ}$. In this limit, the interaction is

$$\mathcal{L} = \frac{\kappa}{\Lambda^{k_U}} O_{SM} O_U \tag{2}$$

with $k_U = d_{SM} + d_U - 4$ and κ redefined accordingly so the action is dimensionless. Since unparticle interactions are heretofore undiscovered, the lower-limit on the energy scale must be $\Lambda_U \geq 1$ TeV, making it an ideal framework for high energy phenomenology.

In an attempt to provide a concrete physical mechanism for such a non-physical phase space, several explanations have been put forth as to the nature of unparticle stuff. These include a composite Banks-Zaks particle with a continuum of masses [50–52], or alternatively a Sommerfeld-like model of massless fermions coupled to a massive vector field [53]. Recently, it was also shown that unparticle-like propagators may be mimicked by a small collection of ordinary particles via Padé approximations [54].

Vector-like unparticle operators O_U couple to baryon currents with dimensionless strength λ according to the interaction

$$\mathcal{L} = \frac{\lambda}{\Lambda_U^{d_U-1}} B_\mu O_U^\mu \quad , \tag{3}$$

which will yield an effective potential of the form [26]

$$V_U(r) \sim \frac{\lambda B_1 B_2}{r^{2d_U-1}} \longrightarrow \frac{\lambda m_1 m_2}{M_B^2 r^{2d_U-1}} \tag{4}$$

where the baryon numbers for the interacting masses are $B_j \approx m_j/M_B$, and M_B is the baryon mass. The modified gravitational potential is then

$$\Phi(r) = \Phi_N(r)\left[1 - \frac{1}{2\pi^{2d_U}} \frac{\Gamma(d_U + \frac{1}{2})\Gamma(d_U - \frac{1}{2})}{\Gamma(2d_U)} \left(\frac{R_{*v}}{r}\right)^{2d_U-2}\right] = \Phi_N(r)\left[1 - \left(\frac{R_v}{r}\right)^{2d_U-2}\right] \tag{5}$$

with the new length scale R_v dependent on the coupling strength λ and the other unparticle parameters. Equation 5 highlights the repulsive nature of vector ungravity, which as we will see is crucial in determining the unique properties of the associated black hole solutions in the relativistic theory.

3 Vector unparticles corrections to the metric

The physical system we are going to investigate is an "hybrid" of classical matter, classical gravity, and "quantum" un-gravity due to the exchange of un-vectors. An initial treatment of the problem has been done in the weak-field, perturbative regime [45,55], but here we present a robust derivation from first principles. The following derivation assumes $d_{BZ} \approx 1$, but departures from this value are considered more extensively in [55].

The action for this system is the sum of a classical functional S_M for matter, and a *non-local* effective action S_U smoothly extending the Einstein-Hilbert action to include un-vectors dynamics,

$$S \equiv S_M + S_U \tag{6}$$

S_M is the classical matter action for a massive, point-like, particle "sitting" in the origin. There is some freedom to choose the explicit form of this functional. Simplicity suggests to introduce S_M in the form of the action for pressure-less, static fluid, with a "singular" (but integrable!) energy density mimicking a "point-mass":

$$S_M \equiv -\int d^4x \sqrt{g}\,\rho(x)\, u^\mu u^\nu , \quad \rho(x) \equiv \frac{M}{\sqrt{g}} \int d\tau\, \delta(x - x(\tau)) \tag{7}$$

The un-gravity action is obtained by combining the Einstein-Hilbert functional and the non-local effective action obtained in [41] :

$$S_U = \frac{1}{2\kappa^2} \int d^4x \sqrt{g} \left[1 + \frac{A_{d_U}}{(2d_U - 1)\sin(\pi d_U)} \frac{\kappa_*^2}{\kappa^2} \left(\frac{-D^2}{\Lambda_U^2} \right)^{1-d_U} \right]^{-1} R \tag{8}$$

where, D^2 is the generally covariant D'Alembertian, which can be treated in the Schwinger representation

$$\left(D^2\right)^{d_U - 1} = \frac{1}{\Gamma(1 - d_U)} \int_0^\infty ds s^{-d_U} e^{-sD^2} , \qquad d_U > 1$$

The coefficient in the numerator of the correction is

$$A_{d_U} \equiv \frac{16\pi^{5/2}}{(2\pi)^{2d_U}} \frac{\Gamma(d_U + 1/2)}{\Gamma(d_U - 1)\Gamma(2d_U)} \tag{9}$$

and κ_* is the coupling between gravity and un-particle. In the vector case

$$\kappa_* \equiv -\frac{\pi}{M_{Pl.}}\left(\frac{\lambda\, M_{Pl.}}{M_B}\right)^2 \tag{10}$$

where, $M_B \sim 1\, GeV$ is the baryon mass. Notice the minus sign taking into account the repulsive nature of the interaction.
As the form of the effective action (8) holds for any kind of unparticle, let us proceed without specifying the coupling constant κ_*, and insert eq.(10) only in the final result.
Our main purpose is to solve the field equations derived from S by assuming the source is static, i.e. the four-velocity field u^μ has only non-vanishing time-like component

$$u^\mu \equiv \left(u^0\,,\vec{0}\right)\,, \quad u^0 = \frac{1}{\sqrt{-g^{00}}} \tag{11}$$

Einstein equations are obtained by varying the action (8) with respect to the metric $g_{\mu\nu}$. By neglecting surface terms coming from the variation of the generally covariant D'Alembertian, we find

$$\begin{aligned} R^\mu_\nu - \frac{1}{2}\delta^\mu_\nu R &= \kappa^2\left[1 + \frac{A_{d_U}\Lambda_U^{2-2d_U}}{(2d_U-1)\,\sin(\pi\, d_U)}\frac{\kappa_*^2}{\kappa^2}(-D)^{d_U-1}\right]T^\mu{}_\nu \\ &\equiv \kappa^2\, T^\mu{}_\nu + \kappa_*^2\frac{A_{d_U}}{\sin(\pi\, d_U)}T_U{}^\mu_\nu \end{aligned} \tag{12}$$

In Eq.(12) we have “shifted” the un-particle terms to the r.h.s. leaving the l.h.s. in the canonical form. As a matter of fact, Eq. (12) can be seen as ‘ordinary” gravity coupled to an “exotic” source term, instead of un-gravity produced by an ordinary particle. The two interpretations are physically equivalent.
The energy-momentum tensor $T^\mu{}_\nu$ is given by [56]

$$T^0_0 = -\frac{M}{4\pi\, r^2}\delta(r) \tag{13}$$

$$T^r_r = 0 \tag{14}$$

$$T^\theta_\theta = T^\phi_\phi = -\frac{M}{16\pi\, r}\delta(r)\,\frac{1}{g_{00}}\partial_r\, g_{00} \tag{15}$$

where, $T^\theta_\theta\,, T^\phi_\phi$ are determined by the requirement $\nabla_\mu T^{\mu\nu} = 0$.
With this kind of energy-momentum tensor the 00 and rr components of the metric

tensor turn out to be of the form

$$g_{rr}^{-1} = 1 - \frac{2}{r} M(r) = -\frac{e^{-h_0}}{g_{00}} \tag{16}$$

where the constant h_0 can be freely re-absorbed into the deviation of the time coordinate, and

$$M(r) = -4\pi \int_r^\infty dr\, r^2 T_0^0 , \qquad r > 0 \tag{17}$$

In Equation (17) the symbol $\int dr$ indicates an indefinite integration. The constant factor e^{h_0} can be safely rescaled to 1 by a redefinition of the time coordinate.

We find,

$$M(r) = \frac{2^{2d_U-2}}{4\pi^{1/2}} \frac{\Gamma(d_U - 1/2)}{\Gamma(2 - d_U)} M\Lambda_U^{2-2d_U} \left(\frac{1}{r}\right)^{2d_U-2} \tag{18}$$

and

$$g_{rr}^{-1} = -g_{00} == 1 + V_N(r)\left[1 - \left(\frac{R_v}{r}\right)^{2d_U-2}\right] \tag{19}$$

$$R_v \equiv \left[\frac{1}{2\pi^{2d_U}} \frac{\Gamma(d_U - 1/2)\Gamma(d_U + 1/2)}{\Gamma(2d_U)}\right]^{\frac{1}{2d_U-2}} \left(\frac{\lambda M_{Pl.}}{M_B}\right)^{\frac{1}{d_U-1}} \Lambda_U^{-1} \tag{20}$$

where, $R_s = 2MG_N = 2M/M_{Pl.}^2$ is the Schwarzschild radius; $V_N(r)$ is the Newton gravitational potential, and R_v is the new gravitational length scale.

The horizon curve is obtained by the condition $g_{rr}^{-1}(r_H) = 0$

$$M = \frac{r_H}{2} \frac{1}{1 - (R_v/r_H)^{2d_U-2}} \equiv M(r_H) \tag{21}$$

The intersections between the line $M =$ const and the curve $M(r_H)$ gives the radii of the inner and outer horizons. In this regard, we notice a first difference with respect the RN metric, where the inner horizon, r_-, can be arbitrarily small. As the mass M is positive definite, we see from Eq.(21) that $r_H > R_v$. That means that the whole horizon curve is shifted to the right by an amount equal to R_v. Thus, r_- can never be smaller than R_v.

If we decrease M the two horizons approach one to the other and finally will merge into the single degenerate horizon of an *extremal black hole*. The mass and the radius of the extremal configuration can be obtained from Eq.(21) and the condition

$$\left(\frac{dM}{dr}\right)_{r=r_e} = 0 \tag{22}$$

Thus, we find

$$r_e = (\, 2d_U - 1\,)^{\frac{1}{2d_U-2}}\, R_v \,, \tag{23}$$

$$M_e = \frac{(\, 2d_U - 1\,)^{\frac{2d_U-1}{2d_U-2}}}{4\,(\, d_U - 1\,)}\, R_v \,, \quad d_U > 1 \tag{24}$$

This result allows us to distinguish three different cases:

(1) $M > M_e$ Massive objects. They are two-horizons black holes
(2) $M = M_e$ Critical objects. They are extremal black hole with a single degenerate horizon
(3) $M < M_e$ Light objects. They would be "naked-singularity" , where no horizon shields the curvature singularity in $r = 0$.

M_e represents the lower bound for the mass of a vector unparticle modified black hole. As we shall see in the next section, the extremal black hole has vanishing Hawking temperature and represents an asymptotic final stage of the evaporation process.

The conditional tense is necessary in the case of light objects, as they have not to be taken too seriously. Indeed, the appearance of a naked-singularity is an alarm signal that the theory we are using is blowing up, rather than a legitimate physical effect. Indeed, invocation of the Cosmic Censorship Conjecture negates the formation of such black holes, and can in fact be used to constrain the unparticle phase space in this situation [55].

Divergence in the Riemann curvature, or tidal forces, at the origin is the unavoidable side-effect of modeling the source of the field as a "point-mass". By packing a finite energy inside a vanishing spacelike volume disrupts the spacetime fabric itself. This is not a physical effect, but it is the due response of a classical theory, i.e. General Relativity, to an unphysical infinite density source. At short distance from the origin General Relativity must be supplemented by Quantum Mechanics inputs in order to provide self-consistent results [57–61]. The whole model we are discussing here, can be trusted only far away from the Planck scale, where, not only matter, but gravity itself must be upgraded to some proper quantum theory.

4 Thermodynamics

Scalar and tensor unparticle-enhanced black hole thermodynamics and their evaporation modes have been addressed previously [46,48,62]. In the case of vector ungravity, the Hawking temperature is

$$T_{d_U} = \frac{1}{4\pi\, r_+ \left[1 - \left(\frac{R_v}{r_+} \right)^{2d_U - 2} \right]} \left[1 - (2d_u - 1) \left(\frac{R_v}{r_+} \right)^{2d_U - 2} \right] \tag{25}$$

by comparing (25 with (23), we see that

$$T_{d_U}\,(r_+ = r_{extr.}) = 0 \tag{26}$$

As it was expected, the extremal black hole has vanishing Hawking temperature. The second zero-temperature configuration is asymptotically approached when $r_+ \to \infty$. Thus, the Hawking temperature increases up a finite maximum value, for $r_{max} > r_{extr}$, and then drops down to zero as $r_+ \to r_{extr.}$

It is interesting to consider the temperature in the two "phases" of the model :
i) *weak-coupling* phase, where $\lambda << 1$, $R_v << r_+$; T_{d_U} takes the standard form

$$T_{d_U} \simeq T_H = \frac{1}{4\pi r_H} \tag{27}$$

ii) *strong-coupling* phase, where $\lambda >> 1$, $R_v >> r_+$; T_{d_U} turns into

$$T_{d_U} \simeq \frac{2d_U - 1}{4\pi r_H} \tag{28}$$

Eq.(28) has the same form as the Hawking temperature for Schwarzschild black hole in D spacetime dimensions

$$T_D \simeq \frac{D-3}{4\pi r_H} \tag{29}$$

It is important to remark that beyond the formal analogy, there is a substantial difference between T_{d_U} and T_D: the topological dimension D is an integer number while the scaling dimension d_U is a *real* number. Thus, in the strong coupling phase, the event horizon behaves like *fractal surface* of *spectral dimension* $d_H = 2d_U$. Let

us elaborate this picture by investigating the Area Law.
We start from the first law of black hole thermodynamics

$$dM = T_{d_U} dS \tag{30}$$

where, $dM = dr_+ \left(\partial M / \partial r_+ \right)$. Equation (30) describes a transformation between two states characterized by a different radius of the event horizon. This transformation is a “path” in the $(M\ , r_+)$ plane along a $d_U = const.$ trajectory.

$$dS = \frac{2\pi\, r_+}{\left[\, 1 - \left(\frac{R_v}{r_+} \right)^{2d_U - 2} \,\right]}\, dr_+ \tag{31}$$

In the weak-coupling phase, unvector contributions can be neglected and Equation (31) takes the standard form

$$dS \simeq 2\pi\, r_+\, dr_+ \tag{32}$$

which gives after integration the celebrated area-entropy law

$$S = \pi\, r_+^2 = \frac{1}{4G_N}\, A_+ \tag{33}$$

In the strong-coupling-phase black hole evolution is different. The key-point is that the final configuration can be, at most, an extremal black hole, but nothing smaller than that. Actually, this configuration is asymptotically approached, as it is $T_{d_U} \to 0$ and smaller and smaller amount of mass is evaporated away. Thus, to compute the entropy (change) from Equation (31) the lower integration limit cannot be smaller than $r_{extr.}$. In this phase, we find

$$dS \simeq \frac{2\pi}{R_v^{2d_U - 2}}\, r_+^{2d_U - 1} dr_+ \tag{34}$$

and

$$S = \frac{\pi\, R_v^2}{d_U} \left[\left(\frac{r_+}{R_v} \right)^{2d_U} - \left(\frac{r_{extr.}}{R_v} \right)^{2d_U} \right] \tag{35}$$

5 Conclusions

We have demonstrated that vector unparticles can modify the Schwarzschild metric for uncharged, unrotating matter, creating a Riessner-Nordström class of solution. The majority of expected characteristics of the resulting black hole – double horizon, extremality conditions and vanishing temperature, *etc*... – are commensurate with the classical case, although we have shown that in the ungravity case there is a minimum (non-zero) inner horizon radius, $r_- > R_v$. The small difference in inner horizon size between the standard RN black hole and un-RN solutions suggests there might be deeper discrepancies in the underlying characteristics. A future path of inquiry might be to investigate the influence of unparticles on the physics of the Cauchy horizon [63,64], which for RN black holes are generally unstable.

The fractal nature of the outer horizon is similar to that obtained for the black holes in [48], furthering the notion that unparticles can increase the effective dimensionality of spacetime by a (non-integer) number of dimensions. A greater understanding of the thermodynamics and decay modes of such black holes can potentially yield observationally-distinct signatures in current or future experiments. Lower mass limits on primordial un-vector black holes have been previously obtained [55], thus if such objects exist in the Universe their evaporation remnants will be visible in this era.

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