2008

The Biot-Savart Law: From Infinitesimal to Infinite

Jeffrey A. Phillips
Loyola Marymount University, jphillips@lmu.edu

Jeff Sanny
Loyola Marymount University, jeff.sanny@lmu.edu

Recommended Citation

J.A. Phillips and J. Sanny, The Physics Teacher 46, 44 (2008); doi: 10.1119/1.2824000

The Biot-Savart Law: From Infinitesimal to Infinite

Jeffrey A. Phillips and *Jeff Sanny,* Loyola Marymount University, Los Angeles, CA

In this paper, we discuss a simple apparatus and accompanying class activity that we have developed to illustrate the Biot-Savart law. Since students in introductory electricity and magnetism courses often find this law a mathematical mystery, we feel that a simple experiment such as this will provide the students a better understanding of the concepts introduced. By collecting data from several finite segment lengths, students are able to infer the $1/r^2$ distance dependence of the magnetic field for infinitesimal segments and the $1/r$ dependence for infinite wires.

The Biot-Savart Law

The Biot-Savart law provides students in introductory electricity and magnetism courses a tool for calculating the magnetic field **B** due to a current-carrying wire of arbitrary shape.[1] The approach used with this empirical law is analogous to that used in the determination of the electric field **E** of a charge distribution of arbitrary shape, normally done earlier in the course. For example, **E** is found by first considering the Coulomb field due to an infinitesimal charge element dq of the distribution and then integrating over the entire distribution:

$$d\mathbf{E}=\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{\mathbf{r}} \quad \text{and} \quad \mathbf{E}=\int d\mathbf{E}. \tag{1}$$

Similarly, in the Biot-Savart law, **B** is obtained by considering the contribution due to an infinitesimal current segment $Id\boldsymbol{\ell}$ of the wire and then integrating over the entire wire:

$$d\mathbf{B}=\frac{\mu_0 I}{4\pi}\frac{d\boldsymbol{\ell}\times\hat{\mathbf{r}}}{r^2} \quad \text{and} \quad \mathbf{B}=\int d\mathbf{B}. \tag{2}$$

This analogy may be carried even further by noting that both the electric and magnetic fields of infinitesimal sources exhibit the same $1/r^2$ dependence on distance from the source. For electric fields, this dependence can be seen using finite charges, providing that they are spherically symmetric.[2] However, the $1/r^2$ dependence in magnetic fields is more difficult to observe as one does need infinitesimal current segments. To overcome this difficulty, we have students analyze the distance dependence for a range of finite segment lengths.

Using the Biot-Savart law one can find the field of a finite straight wire, length ℓ, at a point on the perpendicular bisector of the wire. Here, the field is given by

$$B=\frac{\mu_o I}{4\pi}\frac{\ell}{r\sqrt{r^2+\left(\frac{\ell}{2}\right)^2}}. \tag{3}$$

In the limits $\ell/2 >> r$ and $\ell/2 << r$, the distance dependency of B reduces to $1/r$ and $1/r^2$, respectively, corresponding to the infinite wire and the infinitesimal segment. As will be shown below, it turns out that for intermediate values of ℓ, to an excellent approximation, B is proportional to $1/r^n$, where $2 > n > 1$.

Apparatus

The primary component of our simple apparatus is an 18-gauge solid, insulated wire approximately 1.0 m

DOI: 10.1119/1.2824000

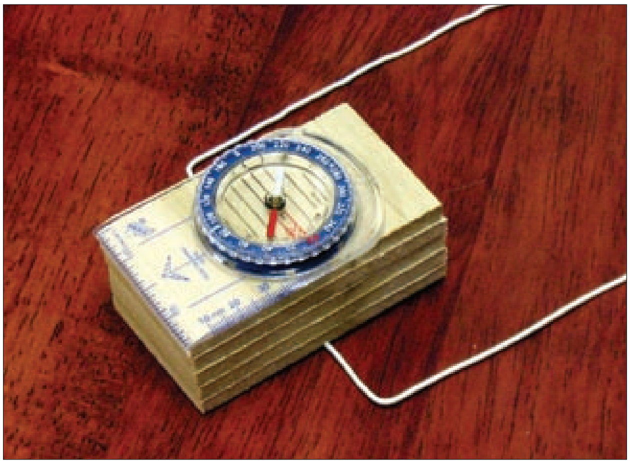

Fig. 1. Photograph of the U-shaped wire, compass, and wood spacers that are used to change the distance between the base segment of the wire and the compass.

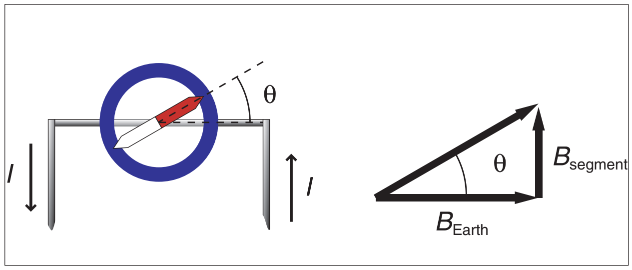

Fig. 2. Schematic of the compass and wire as viewed from above, showing the angle of the compass and its relationship to the sum of the magnetic fields.

in length. The wire is bent into a U-shape, with sharp corners. A compass is used to measure the magnetic field of the base segment of the U as a multiple of the Earth's magnetic field. Different lengths of the base segment can be created by simply changing the location of the bends in the wire.

A high-current power supply is used to produce a current in the wire so that the segment's magnetic field is comparable in magnitude to the Earth's field. To reduce the risk of damaging the power supply, a high-power, low-resistance resistor is connected in series with the wire. In order to determine the magnetic field's dependence on distance, the compass must be placed at different distances from the wire. To do this, small wood spacers, approximately ¼ inch in thickness, were stacked on top of the wire. The bottom block had a small channel carved out that straddled the wire segment, providing a more stable base. A photograph of the configuration is shown in Fig. 1.

Procedure and Data

One can measure the magnetic field of the base segment of the U by placing a compass at points directly above the midpoint of the segment. In order to measure the field as a multiple of the Earth's magnetic field, one has to first align the segment with the horizontal component of the Earth's field, thus eliminating effects from the vertical component and allowing the transition of the segment's magnetic field from approximately $1/r^2$ to nearly $1/r$ to be observed. By having the sides of the U extend perpendicularly from the segment under investigation, the total magnetic field due to the sides will be nearly vertical at the center of the segment, which will not cause a deflection of the compass.

In order to keep the system as transparent as possible, a compass rather than Hall probe is used.[3] First, with the current off, the compass is placed at the desired distance, and the initial angle, which is due just to the Earth's magnetic field, is recorded. Once the current is turned on, the compass needle will deflect to align itself with the total magnetic field. Since the wire segment is parallel to the horizontal component of the Earth's magnetic field, the fields due to the Earth and the wire segment are perpendicular. If the deflection angle of the compass is θ, as shown in Fig. 2, then the magnetic field of the segment and the Earth's magnetic field are related by

$$\frac{B_{\text{segment}}}{B_{\text{Earth}}} = \tan\theta. \tag{4}$$

The distance dependence of the magnetic field of the segment, for a given segment length, is determined by measuring its field at different distances above the midpoint of the segment. The distance between the segment and the compass can be varied by adding or removing wood spacers. Sample data for a 2.0-cm long segment is shown in Fig. 3. By fitting the data with a power law, one determines the value of the exponent n in $B_{\text{segment}} \propto 1/r^n$, which is an excellent approximation of the distance dependence of the magnetic field.

The key to illustrating the distance dependence for infinitesimal and infinite wire segments is to collect data for several different segment lengths. Repeating

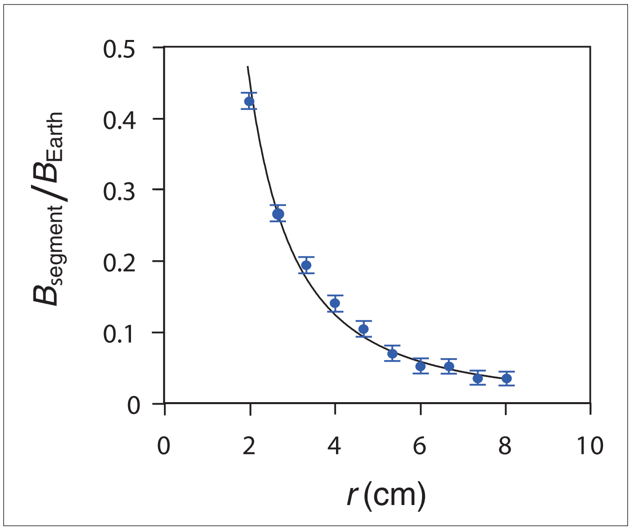

Fig. 3. The magnetic field of a 2.0-cm long wire segment with 4.0-A current, measured as a fraction of the Earth's magnetic field (B_{Earth}). Error bars represent the uncertainty of the compass deflection (±0.5°). The fit line is a power law ($1/r^n$), where n = 1.7±0.1.

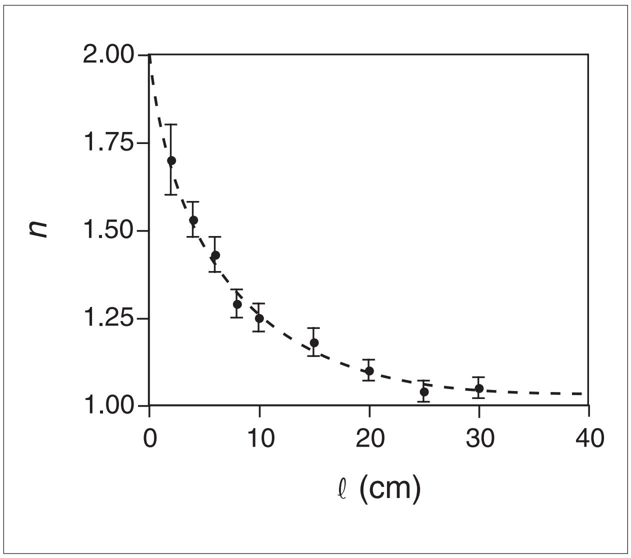

Fig. 4. The variation of the exponent, n, in the power-law distance dependence of the magnetic field as a function of segment length ℓ. As the wire segment approaches an infinitesimal length, the power law approaches $1/r^2$, and in the infinite length limit it approaches $1/r$. The dashed line is provided as a guide for the eye.

the procedure used with the 2.0-cm segment for other lengths yields the distance dependence as a function of segment length and hence a value of n for each case. Figure 4 shows the results for segment lengths between 2.0 cm and 30 cm. The trend clearly approaches n = 1 as the segment length increases, and n = 2 as the length decreases.

Classroom Usage

We have used this activity as an exploration experiment when our students are beginning to study the quantitative relationship between currents and the magnetic fields they produce. At this stage, they have already performed qualitative experiments such as mapping out the direction of the magnetic field around a current-carrying wire using compasses. Hence, they already know that a magnetic field is produced by electric current and the direction of that field. What this activity does is guide students through the distance dependence of the Biot-Savart law.

Each group of students is asked to find the distance dependence for a different wire segment length. As the group collects and analyzes their data, they cannot as yet see any significance in their results. It is only when the groups are asked to share their data that they are able to observe the trend. The instructor can facilitate a discussion of the results, helping the class to make a graph similar to Fig. 4. At this time, students can see the trend of the distance dependence as the segments get smaller. This trend, combined with their previous knowledge of Coulomb's law, should provide them enough evidence to either draw their own conclusions about the infinitesimal segment behavior or readily accept the $1/r^2$ claim from the instructor. Depending on the students' mathematical sophistication, the instructor may decide to either point out the nearly-infinite segment behavior or wait until the students have the chance to integrate the Biot-Savart law to find the result for an infinite wire behavior:

$$B = \frac{\mu_0 I}{2\pi r}, \tag{5}$$

which has the $1/r$ dependence.

Conclusion

We have presented data obtained using a simple apparatus that can be used either as a laboratory or a classroom experiment that motivates a discussion of the Biot-Savart law. The mathematical complexity of this law is often daunting to students in introductory electricity and magnetism courses, and a simple experiment may provide a better understanding of the concepts introduced. The experiment is designed to clearly illustrate the functional form of the Biot-Savart law. The constants cannot be determined without additional measurements, particularly the strength of the Earth's magnetic field. Much like Meiners's apparatus for Coulomb's law,[2] students are able to discover the $1/r^2$ behavior for infinitesimal current segments as well as the $1/r$ behavior for infinite segments.

References

1. J. Sanny and W. Moebs, *University Physics* (McGraw-Hill, Boston, 1997).
2. H.F. Mieners, *Physics Demonstration Experiments* (Ronald Press Company, 1970); K. Cummings, P.W. Laws, E.F. Redish, and P.J. Cooney, *Understanding Physics* (Wiley, New York, 2004).
3. B.J. Pernick, "Simultaneous measurement of the straight wire and the Earth's B field," *Am. J. Phys.* **57**, 90 (1989).

PACS codes: 01.50.Pa, 40.00.00

Jeffrey A. Phillips *is an assistant professor of physics at Loyola Marymount University. His condensed matter interests include nanotribology and liquid helium films. He currently is studying how students' scientific reasoning skills and views about reasoning impact their success in introductory physics.*

Department of Physics, Loyola Marymount University, One LMU Drive, MS 8227, Los Angeles, CA 90045; jphillips@lmu.edu

Jeff Sanny *is a professor of physics and associate dean of the Seaver College of Science and Engineering at Loyola Marymount University. His research is in magnetospheric physics, and he heads an active undergraduate research program that has received funding for nearly 20 years.*

jsanny@lmu.edu