



Digital Commons@

Loyola Marymount University
LMU Loyola Law School

Physics Faculty Works

Frank R. Seaver College of Science and
Engineering

6-1993

Measurement of the microwave dielectric constant for low-loss samples with finite thickness using open-ended coaxial-line probes

G. Q. Jiang

University of California, Los Angeles

W. H. Wong

University of California, Los Angeles

E. Y. Raskovich

University of California, Los Angeles

W. G. Clark

University of California, Los Angeles

W. A. Hines

University of Connecticut - Storrs

Below this page for additional authors: https://digitalcommons.lmu.edu/phys_fac



Part of the [Physics Commons](#)

Digital Commons @ LMU & LLS Citation

Jiang, G. Q.; Wong, W. H.; Raskovich, E. Y.; Clark, W. G.; Hines, W. A.; and Sanny, Jeff, "Measurement of the microwave dielectric constant for low-loss samples with finite thickness using open-ended coaxial-line probes" (1993). *Physics Faculty Works*. 56.

https://digitalcommons.lmu.edu/phys_fac/56

This Article is brought to you for free and open access by the Frank R. Seaver College of Science and Engineering at Digital Commons @ Loyola Marymount University and Loyola Law School. It has been accepted for inclusion in Physics Faculty Works by an authorized administrator of Digital Commons@Loyola Marymount University and Loyola Law School. For more information, please contact digitalcommons@lmu.edu.

Authors

G. Q. Jiang, W. H. Wong, E. Y. Raskovich, W. G. Clark, W. A. Hines, and Jeff Sanny

Measurement of the microwave dielectric constant for low-loss samples with finite thickness using open-ended coaxial-line probes

G. Q. Jiang, W. H. Wong, E. Y. Raskovich, and W. G. Clark
*Department of Physics and Solid State Science Center, University of California at Los Angeles,
Los Angeles, California 90024*

W. A. Hines
*Department of Physics and Institute of Materials Science, University of Connecticut, Storrs,
Connecticut 06269*

J. Sanny
Department of Physics, Loyola Marymount University, Los Angeles, California 90045

(Received 31 July 1992; accepted for publication 8 February 1993)

This work addresses the effect of finite sample thickness on microwave dielectric constant measurements for thin, planar, low-loss samples using the open-ended coaxial-line probe method. Detailed measurements of the dielectric constant were carried out on a wide range of thicknesses of air samples which were backed by infinitely thick teflon and alumina dielectric media. The measurements were made at room temperature for various (50Ω) coaxial-line dimensions, microwave frequencies 4–8 GHz, and power levels near a fraction of a mW. The results provide strong support for previously published theoretical calculations based on a boundary value problem which uses a spectral domain formulation for the aperture fields. From thin, planar samples, values of 10.4 ± 0.5 and 25.9 ± 1.3 were obtained at 5 GHz and 300 K for the bulk dielectric constant of MgO and LaAl_2O_3 , respectively. The applicability of a simple empirical model based on an exponential fit is discussed.

I. INTRODUCTION

Of the various methods which have been used to measure the dielectric properties of materials at microwave frequencies, the so-called "lumped-element" technique has proven to be particularly useful for liquids and biological samples.^{1,2} Recently, this technique has been extended to low-loss solid materials (low loss-tangents, i.e., $\tan \delta < 10^{-3}$).³ In this method, an equivalent circuit consisting of lumped elements is used to relate the impedance of the sensor probe to the (complex) dielectric constant of the sample. A variety of open and closed coaxial-line probes and sample configurations have been employed (for a comprehensive review, see Ref. 4). One such scheme consists of placing an open-ended coaxial line in contact with the sample. A vast majority of the investigations carried out to date, which concern the use of open-ended coaxial-line probes, consider the ideal case of "homogeneous" dielectric samples, i.e., samples which consist of a single dielectric material that is infinitely thick. In this work, we emphasize the case of a thin dielectric sample which is backed by air or some other infinitely thick dielectric medium. To the best of our knowledge, there are only three investigations which have considered the effect of finite sample thickness on the response of open-ended coaxial-line probes.^{3,5,6} In their work concerning layered biological tissues, Anderson *et al.*⁵ considered water layers of varying thickness backed by both air and metal walls. The two configurations, namely, the air and metal walls, represent the lower and upper bounds, respectively, for the change in the fringe-field capacitance from the homogeneous case due to finite sample thickness. In their theoretical calculations, Anderson *et al.*⁵ used the method of moments to

calculate the quasistatic charge distribution over all conductor and dielectric surfaces for a given potential difference between the inner and outer conductors of the coaxial line. Two separate approaches were employed to simulate the presence of the wall numerically. They are described as the finite-wall method and the image coefficients method. From the resulting charge distribution, Anderson *et al.*⁵ calculated the fringe-field capacitance and the electric field around the open end. They obtained good agreement with their measurements on water layers of varying thickness, for both the air and metal backings, and were able to determine the minimum sample thickness and area required to simulate a semi-infinite medium. The treatment of Anderson *et al.*⁵ has not, however, been extended to the general case of a sample with a finite thickness and arbitrary dielectric constant backed by an infinite medium with different dielectric constant. In a later work, Fan *et al.*⁶ considered this general case as a boundary value problem by using a spectral domain formulation for the aperture fields of an open-ended coaxial line. They expressed the axial and radial components of the electric field as well as the angular component of the magnetic field in terms of the angular component of the electric vector potential. By using formulations for the electric vector potential from (circular) antenna theory, and matching the tangential magnetic field at the probe/dielectric boundary, Fan *et al.*⁶ obtained expressions for the electric field components, impedance and, ultimately, the fringe-field capacitance. The expression for the capacitance is in terms of the sample thickness, dielectric constants for the two media, and the dimensions of the coaxial line. Using this expression, they present calculations for water, dioxane, and methanol with

sample thicknesses ranging over almost three decades, backed by both air and metal walls. In terms of the coaxial-line dimensions, Fan *et al.*⁶ also obtained estimates for the minimum sample thickness and area required to simulate a semi-infinite medium. Unfortunately, the only data available until the present study was that of Anderson *et al.*⁵ and Athey *et al.*¹ on water, and neither extends down to samples thin enough to seriously test the theoretical calculations of Fan *et al.*⁶

In this article, we present systematic measurements which were carried out on a wide range of thicknesses of "air samples" which were backed by infinitely thick teflon and alumina (Al₂O₃) dielectric media and compare the results with the theoretical calculations of Fan *et al.*⁶ (We note that when open-ended coaxial-line probes are used to measure the dielectric constant for solid samples, care must be taken to avoid minute air gaps that might exist between the probe and the sample.³ This problem is automatically eliminated with fluid samples.) The measurements were carried out at room temperature for various (50 Ω) coaxial-line dimensions, microwave frequencies 4–8 GHz and power levels near a fraction of a mW. In addition, the procedure was applied to obtain results for two "unknown samples" with finite thicknesses, MgO and LaAl₂O₃. Finally, we note that in our earlier work on solid samples, it was found that a very simple empirical model based on an exponential fit, worked well for the case of samples with a real dielectric constant and a finite sample thickness.³ In terms of the sample thickness, the range of applicability of this model will be discussed.

II. THEORY

In this work, the dielectric constant (real part) for a low-loss sample at microwave frequencies is obtained from a measurement of the phase of the reflection coefficient at the plane interface between the sensor probe and the sample. The probe is an open-end coaxial line having an outer radius b and an inner radius a , along with an infinite ground plane as shown with the bilayered sample in Fig. 1(a). The first layer terminating the open end has a (real) dielectric constant ϵ_1 and thickness d , while the second layer with (real) dielectric constant ϵ_2 extends to infinity. (Since cgs units are used throughout, the permittivity and dielectric constant of a medium are the same.) Using the lumped-element approach, the equivalent circuit for the probe/sample configuration can be represented as shown in Fig. 1(b). With no sample present (coaxial line open to air), the total capacitance associated with the fringe field at the open end, C_p is written as the sum of two capacitances, $C_i = C_f + C_o$ where C_f represents the capacitance due to the fringe field inside of the teflon-filled coaxial line and C_o represents the capacitance due to the fringe field in air. When an infinitely thick sample with (real) dielectric constant ϵ is placed on the probe, C_f is unchanged, C_o becomes ϵC_o and C_i becomes $C_f + \epsilon C_o$. For the sample geometry shown in Fig. 1(a), an effective dielectric constant, ϵ_{eff} , will be defined such that $C_i = C_f + \epsilon_{\text{eff}} C_o$, where ϵ_{eff} will depend on ϵ_1 , ϵ_2 , d , b , and a . It is clear that $\epsilon_{\text{eff}} \rightarrow \epsilon_1$ as $d \rightarrow \infty$ and $\epsilon_{\text{eff}} \rightarrow \epsilon_2$ as $d \rightarrow 0$.

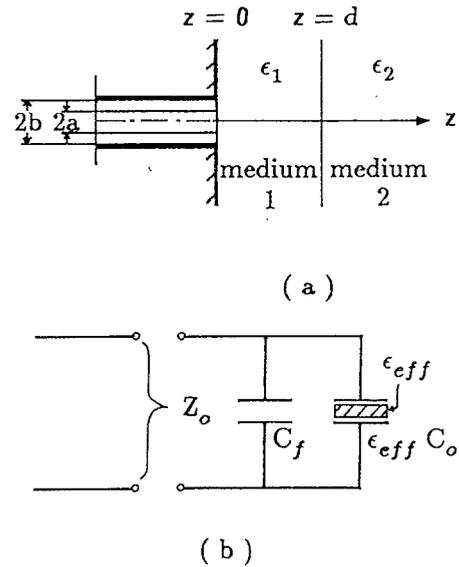


FIG. 1. (a) Sensor probe and sample geometry. Medium 1 has a finite thickness d and dielectric constant ϵ_1 , medium 2 is infinitely thick and has a dielectric constant ϵ_2 . (b) Equivalent circuit consisting of lumped elements. C_f represents the fringe-field capacitance associated with the teflon inside the open-ended coaxial line, C_o represents the fringe-field capacitance associated with the air outside the open-ended coaxial line.

A theoretical calculation for ϵ_{eff} can be obtained from the work of Fan *et al.*⁶ described above. Using a spectral domain formulation, they derive an expression for the fringing field capacitance outside of the coaxial line (C_f is still unchanged) as follows:

$$C = C_1 + C_2, \quad (1)$$

where

$$C_1 = \frac{2\epsilon_1}{[\ln(b/a)]^2} \times \int_a^b \int_a^b \int_0^\pi \frac{\cos(\phi') d\rho d\rho' d\phi'}{[\rho^2 + \rho'^2 - 2\rho\rho' \cos(\phi')]^{1/2}} \quad (2)$$

and

$$C_2 = \frac{4\epsilon_1}{[\ln(b/a)]^2} \sum_{n=1}^{\infty} \left\{ \frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + \epsilon_1} \right\}^n \times \int_a^b \int_a^b \int_0^\pi \frac{\cos(\phi') d\rho d\rho' d\phi}{[\rho^2 + \rho'^2 + 4n^2 d^2 - 2\rho\rho' \cos(\phi')]^{1/2}}. \quad (3)$$

The equations above are Eqs. (7), (8), and (9), respectively, in Ref. 6. It should be noted the C_1 in Eq. (2) above is just the capacitance due to the fringing field outside the coaxial line when the first medium is infinitely thick (i.e., $d \rightarrow \infty$). In terms of our notation, $C_1 = \epsilon_1 C_o$. C_2 represents a correction term to account for the layer of finite thickness. Hence, for finite thickness, the effective dielectric constant, ϵ_{eff} , can be written

$$\epsilon_{\text{eff}} = \epsilon_1 \left\{ 1 + \frac{C_2}{C_1} \right\}. \quad (4)$$

The ratio C_2/C_1 represents a fractional correctional term for the dielectric constant of the first medium. It will be positive when $\epsilon_1 < \epsilon_2$, negative when $\epsilon_1 > \epsilon_2$ and vanishes as $d \rightarrow \infty$. It can be seen that by taking the ratio C_2/C_1 , a common factor of ϵ_1 will cancel; however, the ratio still depends on ϵ_1 , ϵ_2 , d , b , and a . In principle, if the dimensions of the coaxial line (b and a), the dielectric constant of the infinite backing medium (ϵ_2), and the sample thickness (d) are known, it is possible to determine the bulk dielectric constant of the first medium (ϵ_1) from a measurement that yields ϵ_{eff} . It is not practical to solve the equations analytically; however, numerical methods can be employed to obtain useful results (see Sec. III).

III. EXPERIMENTAL APPARATUS AND PROCEDURE

Aside from a few minor improvements, the microwave system and sensor probe used in this work is the same as that described in detail in Ref. 3. All of the measurements presented here involve the reflection coefficient $\tilde{\Gamma} = \Gamma e^{j\phi}$, which is defined at the plane interface between the test dielectric and the sensor probe [$z=0$ in Fig. 1(a)]. It can be expressed as

$$\tilde{\Gamma} = \frac{Z_t - Z_o}{Z_t + Z_o}, \quad (5)$$

where $Z_t = -j/\omega C_t$ is the (complex) impedance of the terminal load, Z_o is the characteristic impedance of the coaxial line, and $\omega = 2\pi f$ with f being the microwave frequency. In terms of the capacitances defined above and the (complex) dielectric constant of the sample

$$Z_t = C_f + \epsilon C_o = C_f + (\epsilon' - j\epsilon'') C_o. \quad (6)$$

For a low-loss material ($\tan \delta \ll 0.1$), $\epsilon'' \approx 0$, $\epsilon \approx \epsilon' = \epsilon$ and $\Gamma \approx 1$. Under these circumstances, the phase of the reflection coefficient, ϕ , can easily be related to ϵ by

$$\phi = -2 \tan^{-1}[\omega Z_o(C_f + \epsilon C_o)]. \quad (7)$$

In order to determine experimentally the values for the fringe capacitances C_f and C_o , as well as establish an absolute value for the phase ϕ , three infinitely thick reference samples are required. Reference 3 describes in detail procedures for calibrating the probe and using a measurement of the phase shift along with Eq. (7) in order to determine the value of ϵ for an unknown sample.

The numerical analysis was carried out on an IBM-PC. In order to greatly reduce the time required for calculating the curve which describes the dependence of ϵ_{eff} on d , a numerical data library was established for each probe. It can be seen that in Eqs. (2) and (3), the triple integrals depend only on the probe dimensions b and a , the sample thickness d , and an index n which runs from one to infinity. It is the evaluation of the integrals which is very time consuming; however, it only has to be done once for a given set of coaxial-line dimensions. In order to determine ϵ_1 , the dielectric constant of the thin unknown material, it

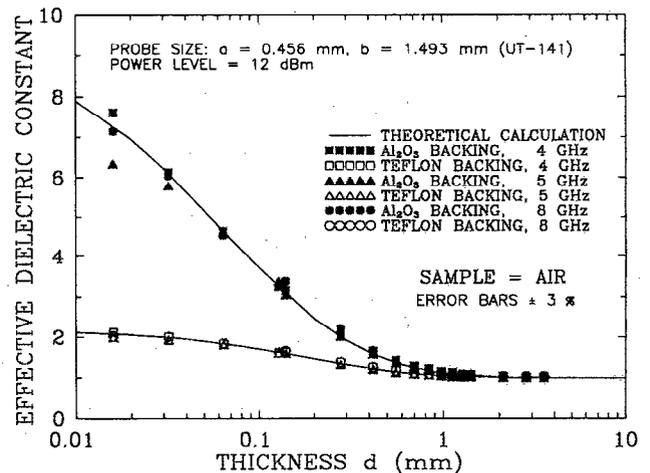


FIG. 2. Measured values of the effective dielectric constant at room temperature ϵ_{eff} vs sample thickness d in mm for air backed by teflon and alumina. The data were obtained for various microwave frequencies using a UT-141 coaxial-line probe and a power level of 12 dBm (0.8 mW). The solid curves are the theoretical calculations using Eqs. (2), (3), and (4) (see text) with $\epsilon_1 = 1.00$ for air, and $\epsilon_2 = 2.20$ and 9.31 for teflon and alumina, respectively.

is only necessary to put in the known value for ϵ_2 , call in the library data and vary ϵ_1 until a best fit of the ϵ_{eff} versus d data is obtained. The computation of the sum in Eq. (3) can be done in a reasonably short time.

IV. EXPERIMENTAL RESULTS AND ANALYSIS

Figure 2 shows the measured values of ϵ_{eff} for various thicknesses of air ($\epsilon_1 = 1.00$) backed by the infinitely thick dielectrics teflon ($\epsilon_2 = 2.20$) and alumina ($\epsilon_2 = 9.31$).³ The probe was constructed from UT-141 semi-rigid microwave coax ($b = 1.493$ mm, $a = 0.456$ mm) and data were obtained for a power level of 12 dBm (≈ 0.8 mW at the sample) at 4, 5, and 8 GHz. The air thickness was controlled by using thin "shims" of kapton and microscope cover glass on the edges of the backing dielectric. The values of the air thickness ranged between 0.016 and 3.5 mm. The curves in Fig. 2 result from the theoretical calculations described in Sec. II and the values of ϵ_1 and ϵ_2 given above. It can be seen that there is good agreement between the experimental data points and the theoretical curves over almost three decades of sample thickness. Also, no significant frequency dependence was observed. In a similar manner, measurements of ϵ_{eff} were made for various thicknesses of air backed by both teflon and alumina using the UT-141 coaxial probe at 5 GHz for two different power levels (-4 dBm = 0.1 mW and 12 dBm = 0.8 mW). Again, good agreement with the theoretical curves was obtained and no significant dependence on the power level was observed. Figure 3 shows the measured values of ϵ_{eff} for various air thicknesses backed by both teflon and alumina using a UT-85 coaxial probe at 5 GHz and 10 dBm (0.5 mW) power level. The smaller dimensions for the UT-85 coaxial probe ($b = 0.836$ mm and $a = 0.255$ mm) made it necessary to establish a new set of library values.

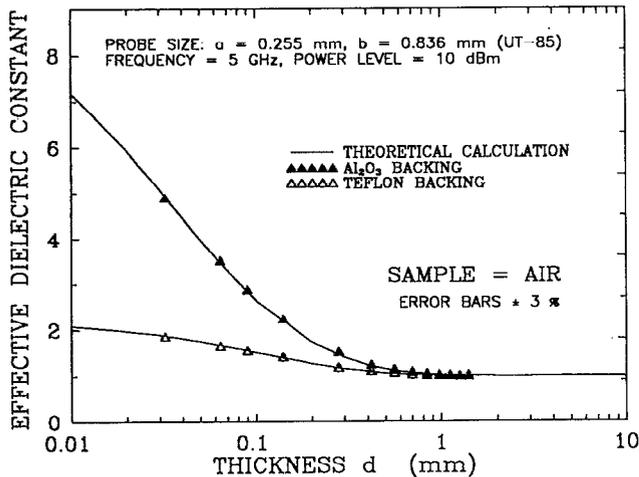


FIG. 3. Measured values of the effective dielectric constant at room temperature ϵ_{eff} vs sample thickness d in mm for air backed by teflon and alumina. The data were obtained using a UT-85 coaxial-line probe at 5 GHz with a power level of 10 dBm (0.5 mW). The solid curves are the theoretical calculations using Eqs. (2), (3), and (4) (see text) with $\epsilon_1 = 1.00$ for air, and $\epsilon_2 = 2.20$ and 9.31 for teflon and alumina, respectively.

Again, good agreement with the theoretical curves is obtained confirming the applicability of the procedure to coaxial lines of various dimensions.

The results described above were obtained for a thin sample of a medium whose dielectric constant is known (air, $\epsilon_1 = 1.00$) and clearly support the theoretical calculations originally proposed by Fan *et al.*⁶ This procedure was used to determine the dielectric constant for thin slabs of two "unknown" dielectrics, MgO and LaAl₂O₃. Figure 4 shows the measured values of ϵ_{eff} for two thicknesses of MgO ($d = 0.328$ mm and $d = 0.452$ mm) obtained with the UT-141 coaxial probe at 5 GHz and 10 dBm (0.5 mW). For both of these samples, air, teflon, and alumina back-

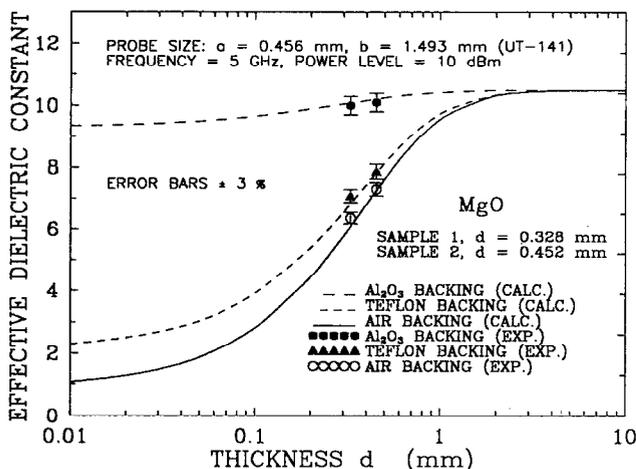


FIG. 4. Measured values of the effective dielectric constant at room temperature ϵ_{eff} vs sample thickness d in mm for MgO backed by air, teflon, and alumina. The data were obtained using a UT-141 coaxial-line probe at 5 GHz with a power level of 10 dBm (0.5 mW). Fitting the data to theoretical curves results in a best value of $\epsilon_1 = 10.4 \pm 0.5$ for MgO.

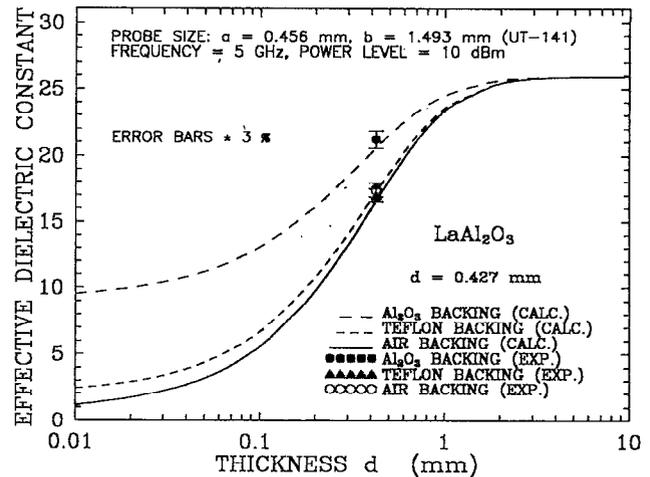


FIG. 5. Measured values of the effective dielectric constant at room temperature ϵ_{eff} vs sample thickness d in mm for LaAl₂O₃ backed by air, teflon, and alumina. The data were obtained using a UT-141 coaxial-line probe at 5 GHz with a power level of 10 dBm (0.5 mW). Fitting the data to theoretical curves results in a best value of $\epsilon_1 = 25.9 \pm 1.3$.

ings were used. By adjusting ϵ_1 , the three curves which best fit the data points were generated resulting a value of $\epsilon_1 = 10.4 \pm 0.5$ for MgO. Figure 5 shows the measured values of ϵ_{eff} for one thickness of LaAl₂O₃ ($d = 0.427$ mm) backed by air, teflon, and alumina, obtained with the UT-141 coaxial probe at 5 GHz and 10 dBm (0.5 mW). From the theoretical fits, a value of $\epsilon_1 = 25.9 \pm 1.3$ was obtained.

V. DISCUSSION

In Sec. IV, we presented systematic measurements of the (real) dielectric constant which were carried out on "air samples" with thicknesses ranging over almost three decades, backed by infinitely thick teflon and alumina dielectric media. Measurements were obtained for various coaxial-line dimensions, microwave frequencies, and power levels. The experimental results were in good agreement with the theoretical work presented by Fan *et al.*⁶ in which they treat the case of a thin dielectric sample which is backed by a second dielectric medium that is infinitely thick by using a spectral domain formulation. The procedure was applied to two solid dielectrics with finite sample thickness, MgO and LaAl₂O₃. MgO is a low-loss dielectric material whose dielectric constant and loss-tangent are well known at room temperature. Our value of 10.4 ± 0.5 for the dielectric constant at 5 GHz is consistent with the literature values which range close to 10 at microwave frequencies.⁷ LaAl₂O₃ has been classified as a perovskite-like compound; its crystallographic properties and structural phase transitions are well studied. It has received considerable attention because of opportunities for application as a dielectric in high-frequency capacitors and magnetohydrodynamic generators. More recently, because of its favorable crystallographic and dielectric properties, it has been proposed as a substrate material for thin films of high T_c superconductors. As such, it could prove to be extremely valuable in the design of superconducting micro-

wave circuit components. Reports of the measurement of the dielectric constant for LaAl_2O_3 at microwave frequencies are quite scarce. One report by Miranda *et al.*⁸ presents measurements carried out in the 20–300 K temperature range at frequencies from 26.5 to 40.0 GHz using a waveguide technique. They obtained a room temperature value of 22 ± 1 which did not change appreciably with frequency. This value decreased by approximately 14% from room temperature to 20 K. Our value of 25.9 ± 1.3 obtained from a thin sample at 5 GHz compares reasonably well.

As can be seen from Eq. (4), the ratio C_2/C_1 represents a fractional correction term for the dielectric constant of the sample due to finite thickness. C_1 , C_2 , and their ratio depend on the dielectric constants of both the sample and backing media, the sample thickness and the probe dimensions as given by Eqs. (2) and (3). It must be noted that these equations were not as successful in calculating absolute values for C_1 and C_2 as they were in calculating the ratio. As described above in Sec. II, $C_1 = \epsilon_1 C_0$ represents the capacitance due to the fringing field outside of the coaxial line when the sample is infinitely thick. Values of C_0 and C_f for a given probe are determined experimentally during the calibration procedure.³ It was found that the value calculated for $C_0 = C_f/\epsilon_1$ using Eq. (2) deviated by as much as 50% from the experimental value. It is fortuitous that the nature of this error is such that it affects C_1 and C_2 by the same common factor as calculations of the ratio agreed with the experimental data within $\pm 3\%$.

In an earlier work on solid samples, it was found that a very simple empirical model based on an exponential fit, worked well for the case of samples with a real dielectric constant and a finite sample thickness.³ The dependence of ϵ_{eff} on d was described by

$$\epsilon_{\text{eff}} = \epsilon_1 + (\epsilon_2 - \epsilon_1) \exp(-d/D), \quad (8)$$

where ϵ_1 and ϵ_2 are defined above and D is an empirical parameter. In the earlier work, the infinite medium behind the sample was taken to be air with $\epsilon_2 = 1.00$. ϵ_{eff} was measured for various thicknesses of a given material and the quantity $\ln_e|\epsilon_{\text{eff}} - \epsilon_1|$ was plotted versus d , with ϵ_1 being treated as an adjustable parameter. In this manner, a value of ϵ_1 was obtained that produced the best linear fit to the data. D was then calculated from the slope of the resulting straight line. In order to explore the applicability of this simple exponential fit, values of ϵ_{eff} were calculated using Eqs. (2), (3), and (4) above for various values of ϵ_1 , with $\epsilon_2 = 1.00$, $a = 0.456$ mm, $b = 1.493$ mm, and $0 < d < 2$ mm. Figure 6 shows the quantity $\ln_e|\epsilon_{\text{eff}} - \epsilon_1|$ plotted versus d over a portion of the range indicated above with $\epsilon_1 = 3.3$, 10.4, and 25.9. It can be seen that within the error which is typical for these measurements ($\pm 3\%$), the exponential fit works quite well over the range $0.1 < d < 1.0$ mm for the three values of ϵ_1 . Deviations occur only for very small thicknesses ($d < 0.1$ mm) and large thicknesses ($d \geq 1.0$ mm). (The a and b dimensions used in the above calculations are appropriate for the UT-141 coaxial-line probe,

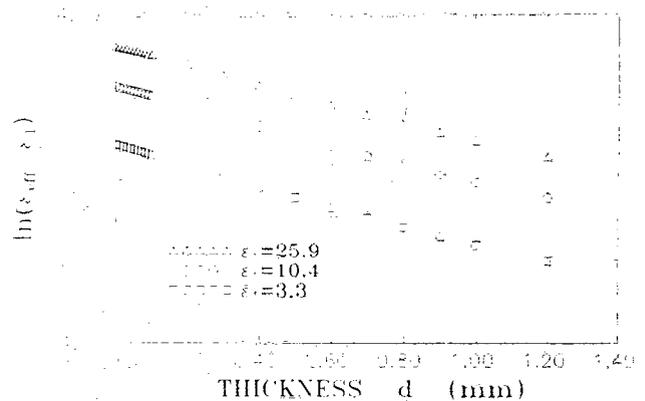


FIG. 6. $\ln_e|\epsilon_{\text{eff}} - \epsilon_1|$ vs sample thickness d in mm, where ϵ_{eff} has been calculated using Eqs. (2), (3), and (4) (see text) for various values of ϵ_1 , with $\epsilon_2 = 1.00$ and the UT-141 coaxial-line probe dimensions ($a = 0.456$ mm and $b = 1.493$ mm): (a) open squares— $\epsilon_1 = 3.30$, (b) open circles— $\epsilon_1 = 10.4$, and (c) open triangles— $\epsilon_1 = 25.9$. The linear behavior over the range $0.1 \text{ mm} < d < 1.0 \text{ mm}$ demonstrates the applicability of a simple empirical fit based on an exponential.

and it is expected that the range of useful d -values would scale with the probe dimensions.) It can be concluded that for low-loss materials with real dielectric constants < 25 , either approach can be used to obtain the dielectric constant at microwave frequencies from measurements made on samples whose thicknesses are such that they cannot be treated as semi-infinite. In situations where several sample thicknesses are available within the range indicated above, the simple exponential fit would be useful. However, in situations where only one or two sample thicknesses are available and/or the sample thicknesses extend outside of the range, it would be more appropriate to use the more detailed calculations based on the work of Fan *et al.*,⁶ and which has been formulated above in Eqs. (2), (3), and (4).

ACKNOWLEDGMENTS

We thank K. Holczer for valuable suggestions regarding the microwave apparatus. This report is based in part upon work supported by the UCLA Center for High Temperature Superconductivity.

¹T. W. Athey, M. A. Stuchly, and S. S. Stuchly, IEEE Trans. Microwave Theory Tech. MTT-30, 82 (1982).

²M. A. Stuchly, T. W. Athley, G. M. Samaras, and G. E. Taylor, IEEE Trans. Microwave Theory Tech. MTT-30, 87 (1982).

³G. Q. Jiang, W. H. Wong, E. Y. Raskovich, W. G. Clark, W. A. Hines, and J. Sanny, Rev. Sci. Instrum. 64, 1614 (1993).

⁴M. A. Stuchly and S. S. Stuchly, IEEE Trans. Instrum. Meas. IM-29, 176 (1980).

⁵L. S. Anderson, G. B. Gajda, and S. S. Stuchly, IEEE Trans. Instrum. Meas. IM-35, 13 (1986).

⁶S. Fan, K. Staebell, and D. Misra, IEEE Trans. Instrum. Meas. IM-39, 435 (1990).

⁷A. R. von Hippel, Dielectric Materials and Applications, (MIT Press, Cambridge, MA, 1954).

⁸F. A. Miranda, W. L. Gordon, K. B. Bhasin, B. T. Ebihara, V. O. Heinen, and C. M. Chorey, Microwave Opt. Tech. Lett. 3, 11 (1990).