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Abstract

Statistics and probability have become an integral part of mathematics education. Therefore it is important to understand whether curricular materials adequately represent statistical ideas. The *Guidelines for Assessment and Instruction in Statistics Education* (GAISE) report (Franklin, Kader, Mewborn, Moreno, Peck, Perry, & Scheaffer, 2007), endorsed by the American Statistical Association, provides a two-dimensional (process and level) framework for statistical learning. This paper examines whether the statistics content contained in the NSF funded elementary curricula *Investigations in Number, Data, and Space*, *Math Trailblazers*, and *Everyday Mathematics* aligns with the GAISE recommendations. Results indicate that there are differences in the approaches used as well as the GAISE components emphasized among the curricula. In light of the fact that the new Common Core State Standards have placed little emphasis in statistics in the elementary grades, it is important to ensure that the minimal amount of statistics that is presented aligns well with the recommendations put forth by the statistics community. The results in this paper provide insight as to the type of statistical preparation students receive when using the NSF funded elementary curricula. As the Common Core places great emphasis on statistics in the middle grades, these results can be used to inform whether students will be prepared for the middle school Common Core goals.

1. Introduction

Statistics and probability have become an important part of K–12 mathematics education in the United States, a change prompted by their inclusion in the National Council of Teachers of

Mathematics (NCTM) recommendations for school mathematics curriculum. The 1989 NCTM document, as well as subsequent documents (National Council of Teachers of Mathematics, 1989, 1991, 1995, 2000, 2006), included a strand dedicated to the study of probability and statistics called *Data Analysis and Probability*. More recently, the *Common Core State Standards* (CCSS), a set of national standards that aim to unify K-12 education across the U.S. for mathematics and English Language Arts, were unveiled in 2010 (http://www.corestandards.org). Although, the NCTM standards placed an increased emphasis on statistics education throughout the entire K-12 curriculum, the CCSS have deemphasized statistics in the elementary grades. In the CCSS, statistical topics are mostly introduced in middle school and continue through high school. In light of this, it is important to examine how existing elementary curricula present statistics in order to ensure that any suggested adjustment or amendments made to fit with the CCSS be done in a way that promotes statistical thinking. Although scarce, the statistics content that students will be exposed to in the elementary grades should promote statistical understanding. It is important to note that the statistical community can help guide the implementation of the CCSS in the classroom. For example, in Groth & Bargagliotti (2012), the authors discuss how the *Guidelines for Assessment and Instruction in Statistics Education: A Pre-K-12 Curriculum Framework* (GAISE) report (Franklin et al., 2007) can be used as a complementary document to the CCSS. If curricular materials are aligned with the GAISE report, they in turn, will be supported by the CCSS.

In response to the promulgation of the NCTM standards, in the 1990s, the National Science Foundation (NSF) supported the development of new mathematics curricula to incorporate NCTM curricular recommendations. At the elementary level, three curricula emerged. *Investigations in Number, Data, and Space* was developed at TERC in Cambridge, Massachusetts (TERC, 2008). The Teaching Integrated Mathematics and Science (TIMS) project housed at the Learning Sciences Research Institute at the University of Illinois at Chicago developed *Math Trailblazers* (TIMS, 2008). The Center for Elementary Mathematics and Science Education at the University of Chicago developed *Everyday Mathematics* (Center for Elementary Mathematics and Science Education, 2008). These curricula will be referred to as *Investigations*, *Trailblazers*, and *Everyday* throughout the text. Each of these curricula approach the teaching and learning of mathematics in a unique way. Overall, the reform curricula illustrate a shift in emphasis from traditional teacher-directed instruction to more student-driven problem solving approaches (Senk & Thompson, 2003).

Several prior studies of student achievement have found positive associations between student performance and the use of the NSF funded curricula (Carter, Beissinger, Cirulis, Gartzman, Kelso, & Wagreich, 2003, Sconiers, 2003, Kehle, Essex, Lambdin, & McCormick, 2007). Some studies on student achievement have focused on particular content strands (Carroll & Isaacs, 2003, Mokros, 2003), however, none have focused on statistics and probability. A critical review of the statistics content in these curricula and how it may relate to student achievement and statistical knowledge is currently lacking from the literature. Before being able to consider achievement, the statistics content presented in the curricula must be reviewed. Because the inclusion of the statistics strand was relatively new when these curricula were originally developed, few guidelines from the statistics community existed to help design and assess their content. The subjects of mathematics and statistics feature important differences (Cobb & Moore, 1997, delMas, 2004, Gal & Garfield, 1997, Rossman, Chance, & Medina, 2006) that must be considered when integrating statistics in a mathematics curriculum.

In 2007, the GAISE report (Franklin et al., 2007) was crafted by the statistics community as a roadmap for statistics learning.[1] This report, endorsed by the American Statistical Association (ASA), describes the manner in which the United States Pre-K-12 curriculum should ensure a statistically literate population. The report defines a statistically literate person as one who is able to formulate questions, collect and analyze data, and interpret results. The document identifies the difference between mathematics and statistics, and offers a three-level scheme, roughly corresponding to elementary, middle, and high school grades, to follow in order to achieve statistical literacy.

Using the GAISE report as a benchmark, this paper examines to what extent the statistics content presented in each of the NSF funded elementary curricula can help produce a statistically literate population. Because the three NSF-supported curricula are compatible with the "focus on active learning" described in the GAISE report (Franklin et al., 2007, p. 13), this paper aims to understand and investigate how these curricula approach and present statistical and probabilistic concepts. In addition, this study responds to the calls by the National Research Council (2004) and Clements (2007) for independent content reviews of existing curricula. Each curriculum is reviewed to understand if and how the guidelines of the GAISE report are met. The following overarching question guides the paper: How well do the NSF funded curricula align with the GAISE report recommendations? It should be noted that all of the CCSS for the elementary grades are encompassed in the GAISE report. Therefore, studying alignment with GAISE, in turn, determines whether the curricula are aligned with the CCSS.

2. Background

2.1. The Role of Curriculum

In the U.S., mathematics curriculum is typically dictated within a state or district by their choices of approved textbooks and standards documents (Reys, Digman, Sutter, & Teuscher, 2005). Although the No Child Left Behind Act (NCLB) legislation and the NCTM standards provides some uniformity of mathematics curricular goals, ultimately much variation exists in the type of content being taught in the classroom (Bargagliotti, Guarino, & Mason, 2010). Specifically with respect to statistics, much variation exists across state curriculum standards (Newton, Dietiker, & Horvath, 2008). Content that makes clear distinctions between mathematics and statistics, scaffolds statistical ideas throughout the grades, and has students actively participating in the curriculum, have been identified by the statistics education community as important components needed to be present in content material to foster statistical learning (Burrill, 2005). This paper investigates and critiques the *intended* curriculum (i.e., the curriculum that is intended to be taught) presented in the NSF funded elementary mathematics textbooks. This paper provides a first and necessary step to understanding the *implemented* statistics curriculum (i.e., the

[1] In 2001 the Conference Board of Mathematical Sciences (CBMS) issued a document called "The Mathematical Education of Teachers." This document included recommendations for the development of teachers' understanding of data analysis, statistics, and probability. The combination of the NCTM and CBMS documents prompted the statistics education community to write the GAISE report (Franklin, C., personal communication, January 1, 2009).

instructional implementation of the intended curriculum) that may be delivered in the classroom using these curricula.

2.2. Elementary Mathematics Curricula

Each of the three curricula being examined in this paper is founded on different principles and ideas. *Investigations* is "organized around key ideas to invite all students into mathematics," *Trailblazers* is founded on the notion that "math is best learned through active solving of real life problems," and *Everyday* focuses on "integrating mathematics into other subject areas" (Education Development Center, 2005, pgs. 8, 10, and 6, respectively). These curricula are widely used across the United States. For example, *Trailblazers* is used by approximately 500,000 students and *Everyday* by over 3,000,000 students. The following sections briefly provide more details about each curriculum and the materials examined in this study.

2.2.1. Investigations

Investigations 2nd edition (TERC, 2008) is a K-5 curriculum based on three mathematics strands: numbers, space, and data. The curriculum is divided into units and the number of units varies for each grade level. Each grade has one unit dedicated to data analysis that is typically taught toward the end of each academic year. A unit is designed to take anywhere from two to eight weeks. A unit is sub-divided into sections called "investigations." The number of investigations per unit varies depending on the grade and topic.

Investigations uses no student textbooks, however, there are student workbooks that include activity sheets for students to complete. Teachers are provided with a Curriculum Unit for each unit in each grade level. These books outline each investigation in a unit, include suggested teacher prompts to ask students, provide assessment activities, and offer professional development teacher support. This study focuses on examining the teacher's Curriculum Units of the second edition of *Investigations* published in 2008.

Four major goals drove the development of the *Investigations* curriculum: (1) present students with meaningful mathematics, (2) emphasize depth in mathematical thinking, (3) communicate mathematics content and pedagogy to teachers, and (4) substantially expand the pool of mathematically literate students (Educational Development Center, 2005). Student learning theory served as the primary guide for the design and presentation of topics in the curriculum. The curriculum was founded on the concept that students come to the classroom with ideas about mathematics and a curriculum must help students develop content knowledge and skills to apply mathematics in different situations. In addition, the curriculum was designed on the idea that teachers are also active participants in the learning of mathematical content and pedagogy. Thus, teachers are viewed as collaborators with students and with the intended curriculum to effectively guide the classroom (http://investigations.terc.edu).

2.2.2. Trailblazers

Math Trailblazers 3rd edition (TIMS, 2008) is a K-5 curriculum founded on the idea of integrated mathematics. The curriculum is developed around problem solving at all grade levels.

In particular, it is described as "integrating mathematics with many disciplines, especially science and language arts" (Education Development Center, 2005).

This curriculum is also organized into units. For each grade level, there are between 16-20 units with material ranging from one week to three weeks long for a unit. For grades 1-5, students have a Student Guide, Discovery Assignment Book, and an Adventure Book. Teachers using this curriculum are provided with a Teacher Implementation Guide that includes a set of tutorials providing content support by presenting background about specific mathematical topics. Three tutorials are specifically related to statistics. They are "Averages," "Estimation, Accuracy, and Error," and "The TIMS Laboratory Method." Teachers are also given a Unit Resource Guide that provides the overview of each lesson, outlines the key content, describes teacher prompts to stimulate student discussion, and includes an identification of the assessment instruments to be used for the given unit. This study focuses on examining the teacher's Unit Resource Guides of the third edition of *Trailblazers* published in 2008.

The main theoretical basis for this curriculum lies in the belief that mathematics should be learned and taught in the context of science. This curriculum originated from the idea of embedding the practice of science in a quantitative framework and the idea of teaching mathematics in a manner that would be meaningful to children (www.mathtrailblazers.com).

2.2.3. Everyday Mathematics

The *Everyday Mathematics* 3rd edition (Center for Elementary Mathematics and Science Education, 2008) curriculum covers grades K-6. It is organized into six content strands that are further subdivided into units and routines. Routines are ongoing activities being followed throughout the academic year. With regard to statistics, this curriculum contains a content strand titled "Data and Chance." *Everyday* focuses on repeated exposure to mathematics topics that build on each other year-by-year. This curriculum employs heavy use of manipulatives as an instructional tool and thus requires a teacher to have a classroom set available. In addition, each child must have access to other objects such as calculators, measuring tools, and drawing tools in order to follow the instruction. Overall, the curriculum is founded on ideas of integrating hands-on experiences and group activities into mathematics lessons.

Students use two journals in every grade of the curriculum as well as an additional Activity Book (grades 1-3), a World Tour Guidebook (4th grade), an American Tour Almanac (5th grade), and a Student Reference Book (available at grades 1-6). Teachers are provided with a manual, a lesson guide, and a resource book. The resource book serves as content teacher support. It describes the background and reasons for including each topic in each unit as well as a review of mathematics content. In general, the support material is geared toward how and why the content material should be taught to students. A comprehensive Teacher's Guide to Activities is also provided. This guide outlines each unit, provides teacher prompts, and includes a description of the topic and activity being done in the unit or lesson. This study examines the Teacher's Guide to Activities of the third edition of *Everyday* published in 2008.

The founding ideas of this curriculum are that students in elementary grades are capable of learning and assimilating complex mathematical ideas. To support such development, a curriculum must make mathematics relevant to their everyday lives by focusing on real-life problem solving. In doing so, the classroom environment should offer large and small group instruction as well as group work and individual activities. Using these different instructional methods, the students use hands-on experiences and games to support their inquiry-based learning (http://everydaymath.uchicago.edu).

2.3. The GAISE Report

Data analysis has become a key component of K-12 mathematics education across the country. For example, the number of students taking AP statistics has increased from 7,500 in 1997 to approximately 142,910 in 2011 (http://professionals.collegeboard.com/profdownload/AP-Student-Score-Distributions.pdf). Although there is a large demand for improved statistics education, this remains the area identified by the Mathematics Education of Teachers Reports (MET I and II, Conference Board of the Mathematical Sciences, 2001, 2012) for which teachers have the largest need in both content and pedagogy. In response to the MET I report, the Teacher Education: Assessment, Methods, and Strategies (TEAMS) conference was held in order to draft a document that makes statistics standards explicit and concrete. This led to the publication of the GAISE report (Franklin et al., 2007). The report's main goal is to provide a fairly detailed guideline about how to achieve a statistically literate high school student at the end of the student's K-12 education. Six statisticians and statistics educators undertook this writing with the help of six other advisors.

The report aims to accomplish two things: it articulates differences between mathematics and statistics and it outlines a two-dimensional framework for statistical learning. One important feature of the framework is that unlike the NCTM standards or any state standards that are outlined by grade, a student's progression is based solely on student experience. In addition, the framework is not defined as a list of topics a student must complete. Instead the report decomposes statistical thinking into four main process components within which a student's level of knowledge progresses.

The report defines a statistically literate person to be one that can formulate questions, collect data, analyze data, and interpret results. In order to characterize the natural advancement of statistical learning, the report describes three-levels of depth, levels A, B, and C, that encompass and outline each process component for each level. The two-dimensional model – process component by level – (Franklin et al., 2007) highlights differences among the levels by the sophistication of the techniques employed to achieve statistical literacy. As the scope of the GAISE report is to provide a comprehensive map of statistics education for grades K-12, we would not expect an elementary curriculum to cover all aspects of the report. In particular, level B ideas may be scarce in elementary curricula and we would not expect to see any level C ideas covered.

Because the GAISE report decomposes statistical ideas by process components and levels, this paper chooses to analyze the alignment between the teacher materials and the guidelines. To achieve this, it is important to examine the questions a teacher is prompted to ask during a lesson, the discussion topics a teacher should introduce during a lesson, and the information the curriculum provides for a teacher. The teacher materials paint a complete picture of the depth these curricula convey. Therefore, this study analyzes how well all the statistics and probability lessons in the teacher materials for each of the three curricula align with the GAISE report.

2.4. Alignment

Several methods for studying alignment in various contexts have been proposed in the literature. Alignment between curriculum and assessment has been discussed in both national (Webb, 1997) and international contexts (McKnight, Britton, Valverde, & Schmidt, 1992). For example, the Third International Mathematics and Science Study (TIMSS) performed an extensive textbook analysis that assessed alignment between curriculum content, curriculum performance expectations, and curriculum perspectives (Robitaille, Schmidt, Raizen, McKnight, Britton, & Nicol, 1993, Schmidt & Houang, 2007). Project 2061 of the American Association for the Advancement of Science (American Association for the Advancement of Science, 1999) assessed whether a curriculum aligned with a set of developed instructional and learning goals. This study's alignment procedure borrows ideas from both the TIMSS study and the Project 2061.

3. Methods

As a first step, our alignment strategy identifies all lessons/units in each K-5 curriculum that contain statistical or probabilistic content. Then, the study determines whether the lessons/units provide the adequate support needed in order for students to achieve statistical literacy by answering four questions:

(1) Which GAISE component(s) is addressed in the lesson?

The GAISE report decomposes statistical content into four main components: formulate questions, collect data, analyze data, and interpret results. A lesson may focus on one or several of the components. This partition of content highlights the different processes a student must learn in order to solve a statistical problem. In order for a curriculum to be aligned with the GAISE guidelines, it should provide several lessons covering each of the components. Students should be given adequate opportunity to familiarize themselves with the different aspects of statistical problem solving. Therefore, as a first measure of alignment, we identify the component or components each statistics lesson falls under.

(2) What GAISE level does the lesson aim to teach?

As students progress through the levels, the statistical techniques, ideas, and methods become more advanced. For example, if we consider the formulate questions component, as a student progresses through its levels, he/she must understand that there are questions that will give deterministic answers and others that will give answers that vary. Only through this level of understanding will the student eventually reach statistical literacy within each component. The curricular material for this type of understanding is primarily exemplified and contained in the teacher probes and the lesson discussion. Thus, in order to evaluate the level at which students are learning the content, our strategy focuses on the material in the curricula specifically for teachers (e.g., teacher prompts, discussion questions, etc.).

(3) What topic within each component and level is being covered?

Once the component(s) and level(s) have been noted for each lesson, then the statistical topic covered is identified. Topics are coded by matching them to the GAISE guideline topics listed in the cross dimension of component and level found in questions (1) and (2).

(4) Is variability introduced when appropriate?

In addition to the four component dimensions, the GAISE report notes the importance of the concept of variability in statistics. In particular, the GAISE report points out that different types of variability exist. Students throughout the levels should be exposed to measurement variability, natural variability, induced variability, and sampling variability. Measurement variability can occur when repeated measures are taken on the same object or individuals but the measurement results from repetition to repetition are not the same. Natural variability refers to the situation where a quality or a characteristic is measured across a population and there is variation in the measurements. Induced variability describes differences among populations driven or caused by a change of a factor or treatment across the different groups. A main focus of modern statistics is to determine the effects of the induced variability while controlling for and accounting for the possible presence of natural variability. When repeated samples of the same size are taken from a population, the sample statistic (e.g., mean, proportion, etc.) will vary from sample to sample. Sampling variability describes this phenomenon. These concepts of variability should be defined in the context of data, data collection, and data generation (see Shaughnessy, 2007, 2008 for more detailed discussion about variability in data). Therefore, each lesson is coded according to whether it covers variability and if so, what type of variability it covers. It should be noted that it is not expected that every lesson cover some type of variability; however, ideally the different types of variability would be mentioned and/or introduced somewhere in each of the elementary curricula. In particular, the GAISE report suggests that measurement, natural, and induced variability be introduced in level A while sampling variability be introduced in level B (Franklin et al., 2007).

Table 1 displays the coding rubric across component, level, and topic. The coding rubric presented in Table 1 is directly adopted from the GAISE report (pp. 23 & 24 for Level A and pp. 37 & 38 or Level B). For example, a lesson that discusses data collection by surveying every student in the class covers content under component “Collect Data” at depth Level A. This lesson would thus be coded as A.2. In addition, the topic a lesson of this type covers is “students conduct a census of the classroom” yielding a final coding of A.2.a.

Table 1. GAISE Levels A and B Coding

Level A	Level B
1. Formulate Questions	1. Formulate Questions
a. Teachers help pose questions b. Students distinguish between statistical solution and fixed answer	a. Students begin to pose their own questions b. Students address questions involving a group larger that their classroom and begin to recognize the distinction among a population, a census, and a sample
2. Collect Data	2. Collect Data
a. Students conduct a census of the classroom b. Students understand individual-to-individual natural variability c. Students conduct simple experiments with nonrandom assignment of treatments d. Students understand induced variability attributable to an experimental condition	a. Students conduct a census of two or more classrooms b. Students design and conduct nonrandom sample surveys and begin to use random selection c. Students design and conduct comparative experiments and begin to use random assignment
3. Analyze Data	3. Analyze Data
a. Students compare individual to individual b. Students compare individual to a group c. Students become aware of group to group comparisons d. Students understand the idea of a distribution e. Students describe a distribution f. Students observe association between two variables g. Student use tools for exploring distributions and association, including: bar graphs, dotplot, stem and leaf plot, scatterplot, tables (using counts), mean, median, mode, range, modal category	a. Students expand their understanding of a data distribution b. Students quantify variability within a group c. Students compare two or more distributions using graphical displays and numerical summaries d. Student use more sophisticated tools for summarizing and comparing distributions, including: histograms, interquartile range, mean absolute deviation, five-number summaries and boxplots e. Students acknowledge sampling error f. Students quantify the strength of association between two variables, develop simple models for association between two numerical variables, and use expanded tools for exploring association, including: contingency tables for two categorical variables, time series plots, quadrant count ratio as a measure of strength of association, simple lines for modeling association between two numerical variables
4. Interpret Results	4. Interpret Results
a. Students infer to the classroom b. Students acknowledge that results may be different in another class or group c. Students recognize the limitation of scope of inference to the classroom	a. Students describe differences between two or more groups with respect to center, spread, and shape b. Students acknowledge that a sample may not be representative of a larger population c. Students understand basic interpretations of measures of association d. Students begin to distinguish between an observational study and a designed experiment e. Students begin to distinguish between "association" and "cause and effect" f. Students recognize sampling variability in summary statistics, such as sample mean and the sample proportion

To code variability, each lesson is carefully studied to determine whether it mentions and introduces the four different types of variability as denoted in Table 2. Because the study examines the teacher materials for each of the curricula, many of the mentions of variability are included as teacher prompts and teacher questions. If the lesson explicitly mentions variability (i.e., the teacher prompts guide students to think about variability) or if the lesson is centered on a specific type of variability, the lesson is assigned the V.1, V.2, V.3, V.4 coding accordingly.

Table 2. GAISE Variability Coding

V.1	Measurement variability
V.2	Natural variability
V.3	Induced variability
V.4	Sampling variability

Two individuals (the author and research assistant) independently coded all of the lessons in two of the grades (kindergarten and fifth grade) of the curricula separately. For these grades, the author and research assistant reviewed and discussed each individual segment of text together in order to reach the final coding. After these discussions, there was complete agreement on all of the coding. For the other four grades (first, second, third, and fourth), the research assistant was not able to participate due to degree completion. For these grades, the author reviewed all segments of text and iteratively coded the lessons. The author examined the text, completed a round of coding, let the coding "rest," and returned to it one month later to start the process again. The purpose of this delay was to give the author a fresh start on the next iteration of coding. The author performed three iterations of coding in this manner in order to reach a final scheme.

4. Results and Discussion

Due to the large number of lessons that were coded for this study (73 lessons for *Trailblazers*, 70 lessons for *Everyday*, 87 lessons for *Investigations*), the coding for the individual lessons is not presented. Individual coding for three example lessons (one for each curriculum) is available online on the JSE website at the following links:

Trailblazers: http://www.amstat.org/publications/jse/v20n3/trailblazers.pdf;

Everyday: http://www.amstat.org/publications/jse/v20n3/everyday.pdf;

Investigations: http://www.amstat.org/publications/jse/v20n3/investigations.pdf.

Tables representing the coding of all other lessons is also available on the JSE website at: http://www.amstat.org/publications/jse/v20n3/coding.pdf.

In this paper, an overall summary for each curriculum is given followed by a discussion comparing the three curricula for each GAISE component. The results focus on summarizing the three curricula's approaches to statistics education as well as the manner in which each curriculum progresses through the grades. The grade level results are synthesized to describe the student opportunity to learn statistics within each of these curricular environments.

4.1. Curriculum Summaries

4.1.1. Investigations Summary

In the early grades (K-2), *Investigations* focuses on having students collect and sort information about their classrooms. To do so, students survey, count, and represent data using graphs and pictures. Several of the investigations have the teacher posing a question followed by the students collecting data in order to answer it. Particular emphasis is placed on sorting data into different categories. For example, in the kindergarten lesson “Favorite Lunch Foods,” a teacher leads students to represent survey information about their favorite foods in a bar graph. Students discuss the best ways and the most appropriate categories (e.g., fruits, drinks, breads) to group the responses. In grade 1, two similar lessons are presented. Students sort shapes according to described attributes of the shapes. Furthermore, students sort different types of buttons according to some specific attribute and represent the frequency of buttons found in each category in a bar graph. The second grade lesson “Favorite Things” also closely mimics what was done in previous grades. In this activity, students answer a survey about their favorite weekend activities and discuss what they can learn from the data on the basis of how the data are sorted. However, this lesson builds on the lessons in the previous grades by having students discuss how different ways of sorting may shed light on different aspects of the data. Although these early grade lessons do not focus on having students draw conclusions from their representations, later lessons in this curriculum do ask students to interpret results.

The *Investigations* curriculum begins to shift from level A to level B as the grades progress. In fourth grade, particular attention is paid to having students formulate their own questions and then interpret their results. Students decide on “what they want to find out” and phrase a two-category survey question to ask the class. In fifth grade, several lessons have students conduct comparative experiments, collect a census of two or more classrooms, and compare and describe differences between two or more groups with respect to the center and the shape of the distribution. For example, students design their own experiment by posing a question that compares two groups. The students carry out the experiment by collecting their data, representing it, and then answering their own research question. In addition, students observe other student’s work and ask questions. In these later grades, *Investigations* places emphasis on the comparison of two groups. Students examine and compare the heights of fourth and fifth grade students. Then, students use the information from this comparison in order to draw conclusions about the height comparisons between 4^{th} grade students and 1^{st} grade students. Students interpret their representations by describing the differences they find between the grade level heights using measures of center and spread.

Generally, *Investigations* introduces and covers statistical topics within a context. When covering a statistics unit, students participate in a series of investigations aimed at building and expanding their statistical knowledge –students are presented with some context, generally crafted with a specific statistical question, and then students engage in an investigation to answer the statistical question. In this manner, students exposed to the *Investigations* curriculum should have a sense of what it means to engage in a statistical thinking process defined by the components put forth in the GAISE report.

4.1.2. Trailblazers Summary

The *Trailblazers* curriculum integrates and incorporates statistics content in several lessons throughout each grade level. Because this curriculum is based on the idea that mathematics should be learned and taught in the context of science, all statistical content is motivated by a question or activity that guides each lesson. As such, *Trailblazers* captures the statistical thinking process put forth in GAISE by simply phrasing and stating all problems within a contextual setting. The curriculum states that teaching this process is "a way to help students learn about the scientific method." In particular, *Trailblazers* introduces and teaches the "TIMS Laboratory Method" that closely aligns with GAISE. The TIMS Laboratory Method is a method taught in *Trailblazers* that students use to organize experiments and investigations. The method involves four components: draw, collect, graph, and explore. In several grades, at least one lesson is specifically dedicated to using the TIMS Lab method to carry out an investigation. For example, in grade 3, students use the TIMS Lab method to study a sample of beans; and in grade 4, students use the method to investigate the relationship between the drop height and bounce height of a tennis ball and a super ball. Because the TIMS Laboratory Method is closely aligned with GAISE, the curriculum presents statistical content as a thinking process very much in the spirit of GAISE.

In the early grades, students using *Trailblazers* learn tally mark tables, pictographs, histograms, and bar graphs. In the later grades, the curriculum focuses on having students use scatterplots and line graphs. For example, in fifth grade, in the lesson "Spreading Out," students investigate the absorbency of paper towels by looking at the relationship between the area formed by a water spot in relation to the number of drops spilled. To do this, students plot points to make a scatterplot. Using the scatterplot, students then begin to discuss how to fit a line to the points.

Overall *Trailblazers* significantly shifts towards depth level B. For example, in fifth grade, the lesson "Searching the Forest," asks students to discuss sampling of different populations and make inferences about the population. In addition to discussing populations and samples explicitly, the lesson directs students to compose a forest population out of tiles. Each group of students has the same forest (i.e., the same tiles) in a bag. Students draw a random sample from their bag and incorporate the information from their samples into a bar graph. Then, using the bar graph, students are instructed to make predictions about the population and recognize that there are differences among their draws. At the most basic level, this lesson hints at how to use probability to make inferences about populations. This lesson exemplifies the advanced depth level that is present in the later grades of the *Trailblazers* curriculum.

4.1.3. Everyday Summary

The *Everyday* curriculum places an emphasis on probability, collecting data in class, representing data in graphs, and computing measures of center. Probability is covered throughout all of the grades with a few lessons dedicated to its study in each grade. For example, in kindergarten, students are asked to look at a tray containing blue and red counters and predict the chances of picking a blue or red counter with their eyes closed. Teachers change the distribution of blue and red counters in the tray to all blue or to all red or to a combination of blue and red. For each distribution, students are asked to predict the color they will draw based on their knowledge of the amount of blue and red counters present on the tray. In first grade, children carry out experiments with a die. Children roll a die and record the results with tally marks. Once they have done this, they begin to speculate whether one number is more likely to be rolled than another. Through this exploration, students discuss probabilistic outcomes and the likelihood of different events.

Another general focus of the *Everyday* curriculum is the representation of data in a tally table, bar graph or histogram. Teachers pose questions to the class (e.g., do you have a pet, how many letters are there in your first and last name) and as a group students create a bar graph to represent the frequency of their responses. In second grade, for example, children count the number of pockets on their clothes in a tally table and then compare the greatest and least number of pockets. Using the pocket data, children make a histogram. Also in second grade, students make a tally table to illustrate their favorite food. The children then transfer the information in the tally table into a bar graph. This process is repeated throughout the grades.

As the grades progress, the focus of analyzing data shifts to finding measures of center. In third grade, 5 lessons out of 15 ask students to compute measures of center; and in fourth grade, 5 out of 16 have students compute measures of center. The main focus of these lessons is to compute the measures of center and then state whether the measure of center can be used to predict a future outcome. Because the focus is on computation, there is only one mention of how to distinguish situations where the mean or median might be more appropriate to use. In third grade, students find the median and the mean for the arm spans and the heights of children and adults. The teacher materials for this lesson include a small summary box located in the margin stating: “Deciding which of them (the mean or the median) will provide the more useful information depends on the situation and how you want to use the results.” The lesson for students, however, stops at computation and does not discuss the possible reasons for computing the mean or median and what may make one better than the other in a given situation. Also, because the lesson merely asks students to carry out computations, there are no comparisons drawn between the adults and children’s arm spans and heights.

In general, this curriculum presents students with statistical tasks such as computing the mean and median or making a graph in a procedural manner. This curriculum emphasizes computation and procedure. The progression of statistical thinking is not very prevalent in the presentation of each lesson, thus a teacher using this curriculum would have to further incorporate the statistical thinking process as described in GAISE.

4.2. Component Summaries

Table 3 summarizes the key features of how each curriculum approaches each component.

Table 3. Key Features for Four Components

	Formulate Questions	Collect Data	Analyze Data	Interpret Results
Investigations	Teachers pose many questions. Students have a chance mostly in the later grades to develop their own questions. All level A frameworks are covered. Level B framework (B.1.a) is also introduced.	Students conduct many classroom surveys and a few surveys where they survey their class as well as other classes. Level A framework is covered and level B begins to be introduced in 4th grade.	Representation of data in graphs and displays is heavily emphasized. Distributions are never introduced explicitly (i.e., the word distribution is never used) but the idea of a distribution and using the shape of the distribution to draw conclusions is introduced. Students are asked to extract information such as range, spread (although done so informally), and center. These ideas are then used as the benchmarks for comparisons. There is a shift to level B in some of the 4th and 5th grade lessons.	Students gain practice in several investigations on how to make group comparisons using center, spread, and shape of the distribution. Students begin to touch upon the idea of a sample being representative of the population. Although there is a shift to level B, not all of level A frameworks are covered. This component shifts towards level B in 4th and 5th grades.
Trailblazers	Explicit questions are not posed by the teacher or students, however, “explorations” of relationships and topics are the drivers of every lesson. The explorations are both guided by teacher posed questions and student posed questions. This curriculum covers all level A and level B.	Students are exposed to numerous settings in which data is collected. All level A topics were covered in this curriculum as well as two of the three level B topics for this component.	Emphasis placed on understanding relationships between two variables. Students are asked to draw bar graphs, look at measures of center, compare distributions, and look at scatterplots. Students spend time studying and comparing the mean and the median as a way to compare distributions. Students using this curriculum will cover all of level A and several level B topics.	Level A topics covered. There is a significant shift to level B to discuss group comparisons, sampling variability, and sample/population relationships.
Everyday	Students are given a task to collect data. Once the data are either collected or provided to the students then the teacher poses questions about it. In this sense, the students are given teacher driven questions to answer. Level A framework covered as well as B.1.a in one kindergarten lesson.	Students collect data within their classroom and while doing experiments with objects such as spinners and cards. They do not collect data from other classrooms or groups even though some of the activities ask the students to make comparisons to other groups outside of the classroom. In these instances, data are either provided for the students or the comparison activity is hypothetical.	There is a real emphasis on using tally mark tables, and computing the mean and the median. Students gain exposure to bar graphs, scatterplots, and line graphs. Numerous lessons throughout the grades are dedicated to computing the mean and computing the median as well as identifying the maximum and minimum of the data.	All lessons have students answer a set of questions about the data at hand. The questions generally ask students to compute a summary statistic. All level A topics are covered as well as two of the level B topics. The emphasis of this curriculum is not to draw inferences and understand the sample/population relationship but focuses on using the data to make predictions about the sample itself.

4.2.1. Formulate Questions

Throughout all of the grades, emphasis is placed on teachers posing questions in all three curricula. All curricula ask students to pose their own questions in one or more lessons throughout the grades. Students using the *Investigations* curriculum get an opportunity to pose their own statistical question one time in each grade. While the development of a question is very guided in kindergarten, as the grades progress the questions become more complex and involved. For example, in kindergarten, students are provided with a sheet entitled "Do you like _____?" on which they must fill in the blank. The names of all students in the class are included on the sheet of paper so that students can track whom they have surveyed. Once their surveys are complete, each student is asked to share the number of people that responded in each category. In fourth grade, students formulate more advanced questions on their own. Students go through an investigation dedicated towards making them think through all of the subtleties of posing their own questions. For example, in the lesson "What Do We Want to Find Out" students pose a question and then collect data that will help answer their question. The lesson imposes the requirement that the question compares two groups. Examples of potential topics for student driven questions are: time it takes to get to school, number of teeth lost, or the amount of TV watched.

In *Everyday*, students are typically given a task to collect some type of data. Once these data are collected then students are presented with a set of questions to answer about the data. In this sense, students are generally given teacher-directed questions, however, the curriculum does not explicitly begin a lesson with a statistical question that then, in turn, motivates the data collection procedure. This differs from the GAISE approach of formulating a question before collecting data. Instead, students begin with a data collection and then questions are posed. Students have one lesson throughout all the grades that is entirely geared toward having students come up with their own questions. Interestingly, this lesson takes place in kindergarten. Teachers instruct students to think of a survey question and write it as a heading on a blank sheet of paper. Then students conduct the survey.

The *Trailblazers* curriculum typically introduces a topic of discussion and then teachers and students pose several questions to guide the studies. Students explicitly pose their own questions in second grade and fifth grade. For example, in the 2nd grade lesson "Undercover Investigation", teachers prompt students to think about the types of questions they can ask about lids (e.g., do plastic lids weigh more than metal lids, is the size of the lid related to its color). As a class, the students must agree on the question they will focus on investigating. In this sense, the lessons are both guided by teacher-posed questions and student-posed questions.

In general, all three curricula cover the level A topics presented in GAISE. The approach in which the Formulate Questions component is introduced and covered, however, differs slightly across curricula. While *Trailblazers* and *Investigations* motivate their lessons with questions, *Everyday* focuses on data collection first and then poses questions about the data collected.

4.2.2. Collect Data

Table 3 illustrates that all three curricula cover all level A topics. The curricula also introduce some level B topics. In *Investigations*, level B lessons begin in 4th grade; in *Everyday* there is one lesson in third grade and one lesson in fourth grade that have B.2.b coding (students design and conduct nonrandom sample surveys and begin to use random selection); and in *Trailblazers*, there is one lesson each year beginning in second grade that has B.2.b coding. For example, the *Trailblazers* fifth grade lesson "Searching the Forest" has students discussing sampling of different populations and beginning to make inferences about the population. The data collection method in this lesson requires students to take repeated random samples thus moving students to understanding random selection.

While in the early grades the curricula focus on conducting classroom surveys, in the later grades, the overall focus shifts to conducting simple experiments and inducing variability in an experiment. In particular, *Investigations* designs a sequence of lessons in "Comparing Balancing Data" around changing the factor in an experiment that is inducing the variability. *Investigations* generally has students survey their own class, however, on a few occasions students survey other classes as well to make comparisons between groups. In this curriculum, each student typically collects his/her own data. On the other hand, in *Trailblazers* and *Everyday*, students typically record information as a class about how each individual performs on an activity or on a personal experience. Although the data each student collects is incorporated into class data making it survey-like, the actual collection procedure differs. Many of the data collection activities in *Trailblazers* and *Everyday* involve students providing the class with their individual response. In this manner, conducting a classroom census is really an assimilation of student individual responses.

Students using *Everyday* collect data within their classrooms through surveys and experiments using objects such as spinners and cards. *Everyday* students do not collect data from other classrooms or groups even though some of the activities and lessons ask students to make comparisons to other groups. In these instances, data are either provided or the comparison activity is hypothetical. The *Trailblazers* curriculum discusses data collection schemes that may induce variability and use random selection (e.g., "Searching the Forest" described above or "Comparing the Lives of Animals and Soap Bubbles"). Because many lessons focus on understanding relationships between two variables (e.g., arm length and height), in several lessons, students make comparisons across different grade levels. In other words, the students discuss how grade level of a student may induce variability in, for example, arm length and height. To do so, students gather data from other classes.

Generally, the three curricula cover level A GAISE topics by having students collect survey data and information about their class. *Everyday* also has students collect data by doing experiments. Level B shifts do occur for the Collect Data component particularly with respect to *Trailblazers* and *Investigations* by asking students to collect data from other groups or classrooms.

4.2.3. Analyze Data

Differences emerge across curricula when comparing the Analyze Data component. Each curriculum emphasizes different aspects of the component. For example, *Everyday* emphasizes using tally marks and computation of the mean and the median. Students describe data distributions by making statements about the most common outcome or the least popular outcome for the class. In addition, students using *Everyday* gain exposure to bar graphs, scatterplots and line graphs. *Everyday* focuses on student exploration of the relationships between two variables and the different types of association between them. This is accomplished by having students plot points on the xy-coordinate plane to examine whether there exists a linear relationship. For example, to explore linear relationships, lesson 10.4 directs students to represent table data on the xy-plane and connect the data points to form a line. Through this exercise, students discuss the concept of a rate as well as how the graph of a line can illustrate and describe the relationship between two variables (coded as B.3.f).

Trailblazers also emphasizes understanding relationships between two variables. Students draw bar graphs, compare measures of center, compare distributions, and study scatterplots. In doing so, students are taught to use these tools to make comparisons, however, contrary to *Everyday*, the comparisons focus on students being able to describe and not compute. Students using *Trailblazers* do compute measures of center, however, the main focus is to learn to use the mean and the median as a way to compare two distributions.

Some level B topics are covered in the curricula, particularly in grades 4 and 5 of *Investigations*. Representation of bar graphs is heavily emphasized throughout the *Investigations* curriculum. Group comparisons are made in the later grades as well. Although, distributions are never introduced explicitly—i.e., the word "distribution" is never used in *Investigations*—the idea of a distribution and using the shape of the distribution to draw conclusions is discussed. Students are asked to extract information such as the range, spread (done so informally), and center. These ideas are used as benchmarks for comparisons. In this manner, *Investigations* leads students to compare two or more distributions by using graphs and numerical summaries.

Overall, the three curricula all align with the GAISE description of Analyze Data, however, their emphases differ. Therefore, a student may develop approaches to data analyses that are dependent on their exposure to a particular curriculum. Some students may become well versed in computation and others may be more comfortable with description.

4.2.4. Interpret Results

Students using all three of these curricula are asked to infer results to their own classroom, however, as the grades progress differences across curricula for the Interpret Results component emerge.

An example of how students are asked to infer results to their classroom is given in kindergarten in *Everyday*. After recording student's favorite colors and representing them in a bar graph, students are asked to interpret the graph by determining the most common and least common answers. Similarly, in the *Trailblazers*' kindergarten unit "Our Homes" students gather information about the types of houses they live in and depict it in a bar graph. Students examine how many students live in each type of home and ask what the most common and least common type of residences are in their class.

As the grades progress, student interpretations are more sophisticated and several lessons in all three curricula shift to level B coding, however, differences are present among the types of ideas emphasized. For example, while both *Investigations* and *Trailblazers* focus on measures of center and spread, their approaches differ slightly. The *Investigations* curriculum places importance on students describing differences between groups using ideas of center and spread (B.4.a) while the *Trailblazers* curriculum focuses on sampling variability in summary statistics of center (B.4.f). *Everyday*, on the other hand, has students compute measures of center but, for the most part, does not ask students to use them as a way to interpret results. There is a distinct shift in the *Investigations* curriculum to level B topics in fourth and fifth grade. Students begin to touch upon the idea of a sample being representative of the population. Although level B topics are introduced, not all level A topics are covered in *Investigations*. Namely, students do not discuss the “limitation of scope of inference to the classroom.”

In contrast to the shift to level B in *Investigations*, the *Everyday* curriculum has little coding for the Interpret Results component. Although scarce, all level A topics are covered in *Everyday* as well as two of the level B topics. It is important to note that the emphasis of this curriculum is not really to draw inference and understand the sample/population relationship but instead to use data to describe the sample itself.

4.2.5. Variability

Variability is explicitly mentioned in *Everyday* and *Trailblazers* but not in *Investigations*. The first mention of variability comes in first grade for *Everyday* (measurement variability), in kindergarten for *Trailblazers* (induced variability), and in fourth grade, though not explicitly, for *Investigations* (sampling variability and induced variability). Throughout the grade levels, there are several opportunities to discuss measurement variability and natural variability (V.1 and V.2) when students represent different distributions, however, there is neither mention of the word variability nor reference to variation as a concept in any of the three curricula. Overall, there are 13 lessons in *Everyday* that touched on some type of variability, 16 lessons in *Trailblazers*, and 14 lessons in *Investigations*.

It is interesting to note that although *Investigations* does not explicitly mention variability of any type and only begins touching upon ideas related to variability in fourth grade, the curriculum does have comparable number of lessons implicitly discussing variability. Induced variability is discussed in fourth grade when comparing heights of first and fourth graders. In this lesson, the investigation is geared at uncovering what could be the reason for the difference in heights between these two groups. In fifth grade, the concepts of measurement variability, natural variability, and induced variability are incorporated throughout the investigations. The first investigation in the curriculum involves students exploring the length of time a person can balance on one foot. Students compare the balance times of students, adults, and other unknown groups (e.g., gymnasts, elderly people, and first or second graders). The factor inducing the variability is thus changing.

In *Everyday*, all four types of variability are explicitly discussed. It is interesting to note that although sampling variability is noted in the GAISE report as being a level B concept, *Everyday* does not elaborate on it in fifth grade, but instead focuses on it in third and fourth. Several lessons throughout *Everyday* implicitly touch upon concepts of variability. For example, in fifth grade, students measure their hand span (i.e., distance from the tip of the thumb to the tip of the index finger when the hand is stretched out). Naturally, different students have different length spans. The class records the span information and represents it graphically in a stem-and-leaf plot. Using the plot, students answer questions about the different length spans and how their span compares. Because there is natural variation among the different spans of the students, this presents an opportunity for the curriculum to define and explore this type of variability. The lesson is clearly interested in uncovering this variation, even though, in this case, the explicit vocabulary is not used.

Trailblazers introduces variability as early as kindergarten. In the kindergarten lesson “Jumpers!”, students touch on the idea of inducing change in an experiment. Students from an older grade are asked to join the kindergarteners for an experiment. The experiment consists of having younger and older students pair together and record how far each can jump. Because the older students are taller, they will tend to jump farther. In this sense, the factor of height induces change in the length a student can jump. Teachers direct the class to compare the results from the older and younger students to reach this conclusion. The characteristic of height of the older students is explicitly mentioned as a reason for why the jump lengths between kindergarteners and older students were different. As the grades progress, *Trailblazers* touches on all of the four types of variability discussed in GAISE, with particular attention paid to measurement variability. For example, six lessons in fifth grade discuss measurement variability (V1). The lesson “How close is close enough” has students estimate the areas of odd shapes on a grid. Irregular shapes such as those with curved sides are given to each group of students. Each student in the group must estimate the area. Because the shapes are irregular, student estimates of the area differ illustrating the presence of measurement variability. The estimates are recorded in a table and the median for each group is computed. Teachers initiate a discussion about how close an estimate has to be in order to be considered good. The class is introduced to measurement error, the concept of margin of error, and they conclude with a general standard to measure closeness.

Generally, *Investigations* pays more attention to induced variability than the other types, *Trailblazers* pays particular attention to measurement variability, and *Everyday* briefly touches on all types.

5. Summary

The GAISE report provides a framework for a student’s statistical learning progression in PreK-12 education. The goal of the report is to create a structure to ensure graduating a statistically literate population at the completion of twelfth grade. To characterize the advancement of statistical learning, the report describes a two-dimensional framework outlining four components (Formulate Questions, Collect Data, Analyze Data, and Interpret Results) and three depth levels (A, B, and C).

In this paper, *Investigations in Number, Data, and Space, Math Trailblazers, and Everyday Mathematics* elementary mathematics curricula were evaluated to discern their alignment with recommendations outlined in GAISE. Findings suggest that all three curricula align well with the topics presented in GAISE. Level A topics are covered for each of the components. The different curricula, however, often have different approaches in their emphasis of topics within the components. In this sense, differences among the three curricula emerge throughout the grade levels. Because of these differences, students using one curriculum may become statistically literate in one component but not in the others.

Additionally, the approach to statistical learning varied from procedural to context driven, with *Everyday* more procedural and *Investigations* and *Trailblazers* more context driven. *Investigations* and *Trailblazers* motivate statistical lessons within context by posing questions. This aligns particularly well with the spirit of the GAISE report. *Trailblazers* defines the TIMS Laboratory method that is perfectly in sync with the spirit of the GAISE report—i.e., viewing statistical learning as an understanding of the statistical thinking process defined by progression through the components. Furthermore, while statistical concepts are built into the mathematics lessons in the *Trailblazers* curriculum, both *Investigations* and *Everyday* separate statistics into different units and strands, respectively. Within a designated statistical unit in *Investigations*, the progression of statistical concepts through the use of the lessons is scaffolded. Students using this curriculum integrate statistical content with other mathematics content.

With respect to the Formulate Questions component, all three curricula align with GAISE and emphasize similar content at similar depth throughout the grades. Teachers help pose questions, and students begin to pose their own questions. *Trailblazers* also includes several lessons geared toward having students make a distinction between statistical solutions and fixed answers. Only in the case of *Trailblazers* do the teacher-directed questions begin to touch on the difference between deterministic and statistical questions. Overall, *Trailblazers* and *Investigations* motivate their lessons with questions while *Everyday* focuses on data collection first and then poses questions about the data collected.

For Collect Data, all students conduct classroom surveys. Generally, the three curricula cover level A topics by having students collect survey data and information about their class. *Trailblazers* and *Investigation* also ask students to collect data from other groups or classrooms. From a statistical perspective, all three of the curricula do not generally push students to compose their own data collection plan given a specific statistical question. The overarching goal of the Collect Data component articulated in the GAISE report is to have students understand how to collect information in order to answer a specific posed question. The lessons in the curricula that include data collection do so by having the teachers instruct students as to what type of data to collect. Teachers could easily shift this emphasis by having students design their own plan instead of providing direct instruction.

For Analyze Data, the three curricula covered most if not all of the level A depth as well as shifting students to level B depth by covering understanding distributions, comparing two or more distributions, and modeling relationships between two variables. The curricula tended to allow students to express their own method of analysis before providing guidance. In several instances, students were asked to organize their data in order to answer a set of questions. These types of activities encouraged students to determine the type of graph, representation, or method they wanted to use in order to answer the posed questions. While performing these tasks, students were encouraged to come up with several different methods of analysis allowing students to think about what strategy they would like to employ in order to best analyze the data at hand. Although there are many similarities among the curricula with respect to this component, the emphasis across the curricula differed. Students using *Everyday* are well versed in computation and students using *Trailblazers* and *Investigations* are practiced in drawing group comparisons.

The curricula have students Interpret Results at level A inferring to the classroom and acknowledging that their results may be different in another class or group. At level B, students in the later grades use measures of center to compare populations. The *Trailblazers* and *Investigations* curricula places great focus on interpreting results as defined in the GAISE report. All of the curricula present students with a set of questions to answer given the data at hand. Students are asked to draw conclusions by consulting with the analysis procedures they performed.

6. Conclusion

The findings and results presented through this study reveal that the NSF funded elementary curricula differ with respect to their alignment with the GAISE guidelines. Considerable variation was found along the content dimensions and the general approach to statistical learning. The different theoretical assumptions used to design the different curricula may partially explain the choices the authors of the curricula made in formulating the statistics and probability activities. For example, because the *Trailblazers* curriculum was grounded in the idea of embedding the practice of science in a quantitative framework and the idea of teaching mathematics in a meaningful context, statistical ideas emerge very naturally. Having students work through problems that are set in a scientific context may provide an easy setup for posing statistical questions, designing data collection plans, and analyzing, answering, and interpreting results.

In general, there is an overall lack of a statistical point of view found in these curricula. This may be due to the authors' mathematical mindsets. For example, while students using *Everyday* work frequently through probabilistic concepts, the lessons mainly focus on probability as a theory (e.g., likelihood of events, frequency of particular outcomes) instead of discussing probability, for example, in the context of sampling. Ideally, mathematical probability *and* statistical probability would both be introduced in a curriculum. Slight refinements to the curricular material to include a more statistical point of view may prove to be a useful way to help students reach statistical literacy. As such, future revisions of the curricula may want to differentiate between mathematical probability and statistical probability, introduce the different types of variability explicitly, motivate all lessons from a posed statistical question, pose statistical questions instead of deterministic questions, and overall adapt a statistical point of view when covering statistics topics. It should be noted that the GAISE framework was put forth after the first versions of these curricula were published. In light of this fact, a very positive finding of this study is that much of the curricula are in line with the statistical learning framework. This suggests that the general approach of the reform curricula does follow the guidelines for statistical learning put forth by the statistics community.

The authors of the CCSS did not include statistics in elementary school because they followed the recommendations of the other recent documents such as the Foundations for Success: The Final Report of the National Mathematics Advisory Panel (NMAP, 2008) and Curriculum Focal Points (NCTM, 2006) (McCallum, W., personal communication, November 3, 2010). The CCSS do contain a significant amount of statistical content but concentrated over fewer grades. Their idea was for elementary mathematics to focus on number concepts that in their view will serve as a basis for statistics (McCallum, W., personal communication, November 3, 2010). In addition, although the committees assembled to write the National Math Panel Report, the Curriculum Focal Points, and the CCSS include top-notch math educators and a handful of excellent mathematicians, only a few statisticians and statistics educators participated. While some of the statisticians' and statistics education specialists' suggestions were incorporated into the CCSS at the middle and high school level, at the elementary levels they largely were not. This ultimately results in a lack of statistical voice present in the policy document at the elementary level. Since the statistics community has embraced the GAISE framework as providing an accurate and helpful progression for the learning of statistical content, these guidelines should be considered when discussing mathematics and statistics education in the U.S. After all, the motivating factor for writing the GAISE framework was to provide the education community with guidelines for statistics learning from the statistics community. Thus, the adaptation of new standards, other guidelines, and curricula should include statistical points of view.

In order to gauge the effect of the differences among the elementary curricula, future research is needed to tie these curricula to student learning of statistical concepts. Furthermore, future research examining and validating whether the GAISE report aligns with student learning would be worthwhile.

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