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Recommendations to Internal Auditors Regarding the Auditing and Attestation of Mathematical Programming Models

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RECOMMENDATIONS TO INTERNAL AUDITORS REGARDING THE AUDITING AND ATTESTATION OF MATHEMATICAL PROGRAMMING MODELS

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ABSTRACT

Mathematical programming planning models increase operational efficiency and minimize operating costs, but the underlying mathematics generally is complex. Combinatorial optimization is technically sophisticated which requires a strong quantitative background to successfully implement. Most internal auditors will not have the technical training to critically assess the underlying mathematics of mathematical programming planning models, but the internal auditor can still provide insight and attestation which can increase the efficiency of mathematical programming planning models.

Keywords: internal audit; mathematical programming planning models; combinational optimization; minimum cost network flows; economic order quantity

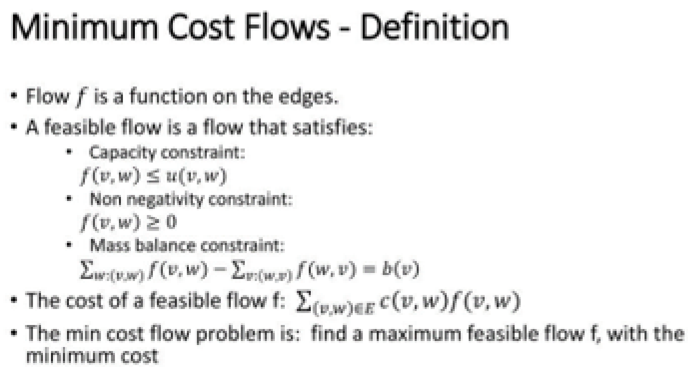
INTRODUCTION

The decision sciences are essential techniques for the entity which wishes to optimize its business processes. Mathematical programming planning models such as *minimum cost network flows* or the *economic order quantity* are used extensively by many entities to maximize operational efficiency and/or minimize operating costs. Package delivery services such as UPS and DHL use modified linear programming algorithms to develop the delivery route for each of their drivers (although these software programs are proprietary to the company). The more accurate the software calculations, the more efficiently their drivers can make their deliveries. Another application is the growing and evolving food delivery services such as *DoorDash*. *DoorDash* has over 10,000 employees, which implies they have some complex routes which would necessitate the use of mathematical programming planning software. *DoorDash* also makes bicycle deliveries which injects an additional layer of planning complexity.

Heuristics could be employed to determine a good feasible solution that may not be optimal but still represents an excellent low-cost solution.

The *minimum cost network flows* or the *economic order quantity* calculations could be computationally complex and require a specialist to formulate and calculate. The generalized mathematical formulation of a *minimum cost network flows* follows:

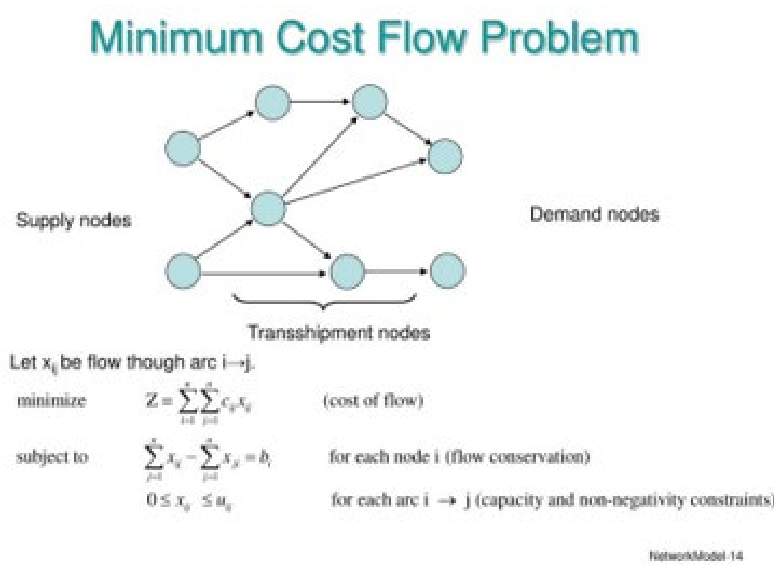
Figure 1
Minimum Cost Flow- Definition



Note: Adapted from Minimum Cost Flows: Network Simplex Algorithm PowerPoint, by L. Teller, slide 3. Web 2014 by SlideServe.

Route planning utilizes mathematical programming to determine the optimal delivery route for the drivers. If the delivery route has dozens of delivery destinations, the combinatorial optimization could have over 10,000 feasible solutions although only a few will be optimal.

Figure 2
Minimum Cost Flow Problem



Note: Adapted from Network Optimization Model PowerPoint, in SlideServe, slide 14. Web 2013 by SlideServe.

This could require substantial computing resources if there are many possible feasible combinations.

LITERATURE REVIEW

The literature review of this topic is sparse. There are many articles on internal auditing and many on mathematical programming, but very few on the intersection of these two topics. The first two articles mentioned in the literature below mentioned the intersection of these two topics, but they had a different focus than this paper. The authors of both these papers were from universities in Europe.

Andrade (2020) conducted a survey of the audits of mathematical programming models. The author summarizes the surveys and categorizes the surveys based upon what survey respondents provided. It is an excellent summary table, but this paper focuses on what was done without going into any in-depth analysis of how the audit was conducted.

Drogalas, Petridis K., Petridis N. and Zografidou (2020) examine mathematical programming planning models for local government organizations. They performed regression analysis on the different software they studied and used R^2 (goodness-of-fit) as the primary metric to measure the accuracy of the model studied. Again, the study was only on local government organizations so their conclusions may not be applicable to other entities, especially businesses.

The *Institute of Internal Auditors* (2022) explains "why a clear understanding of roles is critical to effective compliance and independent assurance." This paper will attempt to list the bounds of team members and why collaboration is important in the internal audit of mathematical programming planning models. The IIA paper goes further and explains the necessity of "clarity and collaboration for stronger governance."

Smith (2012) constructed a checklist of 65 tasks for an internal audit that is an industry standard. This checklist can be used at the beginning of the internal audit since it has all the tasks needed for a complete audit. Not all of these tasks are necessary for the internal audit of mathematical planning models and unnecessary tasks can be trimmed from this checklist. This checklist brings standardization into the internal auditing process now and into the future.

Vincente (2023) of the Auditboard.com provides a contemporary audit checklist that provides step-by-step recommendations. This audit checklist is general and applies to any internal audit so provides a foundation for the internal audit of mathematical planning models. Specific audit procedures can be implemented into this general guideline to conform to Generally Accepted Auditing Standards. The author suggests the use of subject matter experts to assist with an internal audit.

APPLICATIONS OF INTERNAL AUDITING

Internal auditing is employed to determine the accuracy of mathematical programming planning models or to identify any potential risks. If the parameters of a mathematical programming planning model are stochastic, internal auditing is necessary to determine if the results are consistently optimal.

Because the level of mathematics in a mathematical programming planning model is beyond the scope of many internal auditors, an initial reaction would have the subject matter expert or software developer self-audit the mathematical programming planning model. This would represent a conflict of interest, however, because the subject matter expert or software developer may not be objective or unbiased conducting the audit. Also, the subject matter expert or software developer might not be well-versed in Generally Accepted Auditing Standards and could not conduct a feasible audit.

A third-party mathematician or operations researcher could be recruited to assist with the audit of mathematical programming planning models, but again these individuals may not have the requisite auditing expertise. If a mathematician or operations researcher will assist with an internal audit, they should work closely with the internal auditor so that the audit can be attested to.

While the internal auditor may lack the background to audit the mathematics of a mathematical programming planning model, the internal auditor can still provide a valuable function. Statistical tests could be employed to determine the statistical accuracy of the mathematical programming planning model. A Certified Internal Auditor will have a good working background with sampling theory and statistics and could employ statistics in the audit.

NECESSITY OF AN INTERNAL AUDIT

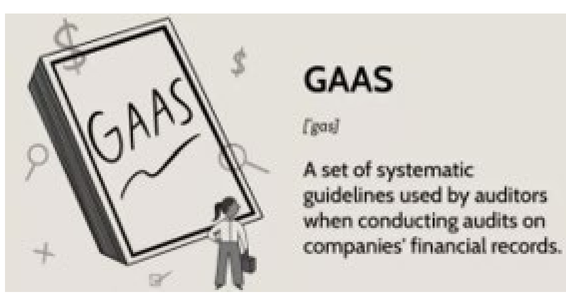
Many individuals of an entity could be against the internal audit process because they feel internal audits are intrusive and a form of micromanagement. They don't see the value of an internal audit and incorrectly see it as a potential to get into trouble. If done correctly, an internal audit represents "preventive medicine" and can prevent a process from getting out of control.

The engagement in this internal audit should foster a spirit of collaboration and should be non-confrontational. The goal of this internal audit is to increase operational efficiency, not an error-finding mission. This team should develop metrics that are testable and standardized. The team should construct this internal audit as a long-term, continuing, permanent audit as long as a given mathematical programming planning model is employed. An internal audit may be triggered if there is a substantial change to the algorithm.

The internal auditor needs to perform the internal audit "by the books" and be a good teammate simultaneously. It is imperative that the internal auditor practice good "people skills" and conduct this internal audit smoothly. The internal auditor should follow Generally Accepted Auditing Standards very closely to ensure that the internal audit is complete.

Figure 3

Generally Accepted Auditing Standards (GAAS): Definition



Note. Adapted from Generally Accepted Auditing Standards: Definition, GAAS vs. GAAP by A. Tuovila. Web 2023 by Investopedia.

Figure 4
Generally Accepted Auditing Standards (GAAS)

Note. Adapted from Modern Auditing: Assurance Services and the Integrity of Financial Reporting, 8th Edition, by W. C. Boynton and R. N. Johnson in SlideServe, slide 8. Web 2014 by SlideServe.

KirkpatrickPrice is a firm which specializes in cybersecurity and compliance audits. They are an industry leader with a variety of professional certifications and licenses, including one from the *Institute of Internal Auditors*. KirkpatrickPrice lists the five reasons for the importance of an internal audit:

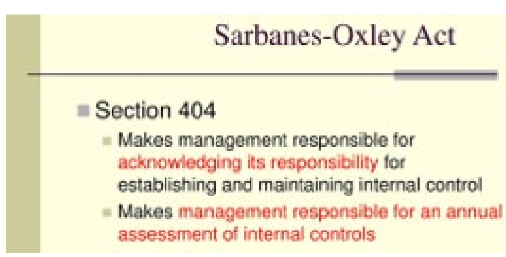
- Provides objective insight
- Improves efficiency of operations
- Evaluates risks and protects assets
- Assesses organizational controls
- Ensures legal compliance

Each of the reasons are important, but legal compliance should be a point of emphasis for any entity (Holladay, 2023).

The *Sarbanes-Oxley Act of 2002* clearly states the necessity of internal controls and management responsibility. Supply chain management and inventory management are expensive and require large expenditures so it is imperative that cost minimization methods are implemented. If adequate internal controls are not in place, cost overruns from applying these models could be substantial. In many instances mathematical programming planning models are employed by an entity to determine the minimum cost solution so internal controls must be in place to check if these are meeting its goals.

Section 404 of the *Sarbanes-Oxley Act of 2002* addresses the internal control requirement:

Figure 5
Sarbanes-Oxley Act

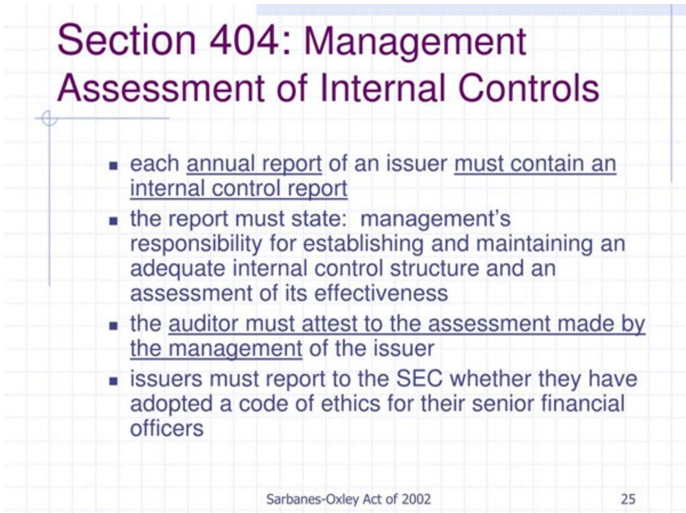


Note. Adapted from Assessing IT Controls under Sarbanes-Oxley by C. Delano Gray, slide 18. Web 2011 in SlideServe.

The *Sarbanes-Oxley Act of 2002* is wide-ranging and is not limited to financial statement analysis only. A comprehensive plan to audit mathematical programming planning models is a key to ensure compliance.

Figure 6

Section 404: Management Assessment of Internal Controls



Note. Adapted from Sarbanes-Oxley Act of 2002, in SlideServe, slide 25. Web 2012 by SlideServe.

An area to monitor closely is change management. These mathematical programming planning models deal with changing objectives on a daily basis so the mathematical programming planning models must be executed daily. An important point to remember is that the input parameters are not static. A package delivery example would be a road that is suddenly closed for whatever reason. This closed road is now eliminated from the feasible solution and an adjustment must be made to the mathematical programming planning model algorithm.

The adjustment to the underlying algorithm may not be trivial and could require many hours of coding to implement this change. A rudimentary formulation illustrated below shows the simple task of assigning buses to minimize passenger wait times:

Figure 7

Mathematical Formulation of Dynamic Network Flow Problem SBP

$$
\begin{aligned}
&(\text{SBP}) \ \min \left(\sum_{v \in V_S} \sum_{t \in [T]} x_{v,v}^{t,0} \right) && \text{total waiting time} & \\
&s.t. \ x_{s,v}^{t,0} = p_v^t && v \in V_S,\, t \in [T] & (1)\\
&\sum_{v \in V_S} \sum_{t \in [T]} x_{s,v}^{t,1} = B && & (2)\\
&x_{v,v}^{t,1} \leq b(v) && v \in V, t \in [T] & (3)\\
&\sum_{(v,w) \in A} x_{v,w}^{t-\tau(v,w),l} = \sum_{(v,w) \in A} x_{v,w}^{t,l} && w \in V \setminus \{s,d\},\, t \in [T],\, l \in \{0,1\} & (4)\\
&x_{d,v}^{t,0} = 0 && v \in V_S, t \in [T] & (5)\\
&C \cdot x_{v,w}^{t,1} \geq x_{v,w}^{t,0} && (v,w) \in A,\, v \in V \setminus \{s\},\, t \in [T] & (6)\\
&x_{v,w}^{t,k} \in \mathbb{N}_{\geq 0} && v, w \in V,\, t \in [T],\, k \in \{0,1\} & (7)
\end{aligned}
$$

$x_{v,w}^{t,0} \in \mathbb{N}_{\geq 0}$	number of passengers traveling from v to w at time t
$x_{v,w}^{t,1} \in \mathbb{N}_{\geq 0}$	number of buses traveling from v to w at time t

Note. Adapted from an article titled; A hybrid and multiscale approach to model and simulate mobility in the context of public events (2016) by D. H. Biedermann et al., published in the journal, *Transportation Research Procedia* on p. 357.

One change could have a “ripple effect" to many parts of the formulation and the algorithm must be adjusted accordingly. Change management cannot be handled on an ad hoc basis but rather should follow a standardized set of procedures which can accept almost every type of change. This set of procedures should be clear and logical to audit.

Mathematical programming planning models must deal with change on a continuing basis. Change management is crucial and the internal auditor must determine if change management is controlled and consistently follow the rules and procedures of change management for that entity. There is a high level of risk if the change management process is not controlled. Some key auditing questions would be:

- Are change management procedures standardized?
- Who approves and is responsible for the change?
- What documents are required for change?
- What is the documentation process for change management and is this process followed and enforced?

There are many other pertinent questions the internal auditor must address to ensure that changes don’t lead to processes which are not in control. Change is an inherent part of mathematical programming planning models so control mechanisms must be in place to regulate the change management process. An internal audit should be initiated if there is a substantial change to the mathematical programming planning model algorithm.

Figure 8
Change Management Workflow

Note. Adapted from Change management by S. Jahan. Web 2020 in Human Recourse Management Practice.

Another area of focus should be the adherence to the directives from the mathematical programming planning models. Do the delivery drivers exactly follow the route calculated by a *minimum cost network flow model*? Does a procurement department exactly follow the number of units recommended and the order schedule from an *economic order quantity* model? This may not be a problem if procurement is automated in an *enterprise resource planning* system but may become an issue if procurement is still mostly manual for an entity.

For a mathematical programming planning model to be truly effective, the workers must follow the directives of the mathematical programming planning models as closely as possible. A time and motion study is used to analyze work efficiency through the observation and timing of tasks. Time-motion analysis and internal audits should be conducted on individuals involved or the results from GPS tracking should be examined to determine the level of adherence to the directives of the algorithms.

APPROPRIATE TEST STATISTICS

While the internal auditor may lack the background to audit the mathematics of a mathematical programming planning model, the internal auditor can still provide a valuable function. Statistical tests could be employed to determine the statistical accuracy of the mathematical programming planning model. A Certified Internal Auditor will have a good working background in sampling theory and statistics and could employ statistics in the audit.

The Z-test, the student's t-test, F-test, ANOVA and other statistical tests are standard tools which have proven effectiveness. These tests provide insight into a process under the right circumstances. Regression analysis and ANOVA are robust testing methodologies where the statistical software provides results in a standard fashion for ease of use and interpretation.

A key assumption of the statistical tests mentioned above is that the underlying variables are normally distributed. If the normal distribution assumption does not hold then use of the test statistics above could lead to incorrect conclusions. The underlying probability distribution should be determined by an expert in probability theory and is therefore out of the area of expertise of many internal auditors.

Inferential statistics may be inapplicable to the audit of mathematical programming planning models but descriptive statistics can provide valuable insight into the accuracy of mathematical programming planning models. Fundamentally mathematical programming planning models are forecasts so the same metrics used to assess the accuracy of time-series forecasting model such as moving average and exponential smoothing are applicable to mathematical programming planning models. These descriptive statistics could include the mean average deviation (MAD), mean squared error (MSE) and mean absolute percentage error (MAPE).

The mean average deviation (MAD) and mean squared error (MSE) have a characteristic which could make them difficult to apply to an internal audit of mathematical programming planning models.

Figure 9
Mean Square Error – Example

Mean Squared Error - Example

Period	Actual Demand	Forecasted Demand	Error	Squared Error
7	48	52.69	-4.69	22
8	45	51.15	-6.15	37.82
9	47	49.13	-2.13	4.54
10	45	48.43	-3.43	11.76
11	40	47.31	-7.31	53.44
Total				129.56

$$MSE = \frac{129.56}{5} = 25.91$$

35

Note. Adapted from Supply Chain Management: Demand Planning PowerPoint, in SlideShare, slide 35. Web 2015 by SlideShare.

Both the mean average deviation (MAD) and mean squared error (MSE) provide a single number result which is difficult to interpret by itself. The previous example calculated an MSE of 25.91. Is this a good result or a bad result? This could be low or high but would be highly dependent on the inputs into the equations. This difficulty applies equally to both the mean average deviation (MAD) and mean squared error (MSE). A series of MAD or MSE results are necessary to interpret the information provided by the MAD or MSE results. A trend could be developed but this is a shortcoming of a single MAD or MSE result – a stand-alone result cannot be interpreted without additional references.

An appropriate test statistic is the mean absolute percentage error (MAPE), which is calculated by the following formula:

Figure 10
Mean Absolute Percentage Error (MAPE) Formula

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{|y_i - \hat{y}_i|}{y_i}.100\%$$

Note. Adapted from Performance Metrics: Regression Model by Diwas. Web 2020 in Bishal's Blog.

Lewis (1982) provides a table which classifies MAPE results and provides insight into the accuracy of a forecast (p. 40):

Figure 11
MAPE Interpretation

MAPE	Interpretation
<10	Highly accurate forecasting
10-20	Good forecasting
20-50	Reasonable forecasting
>50	Inaccurate forecasting

Source: Lewis (1982, p. 40)

Note. Adapted from an article titled; Using the R-MAPE index as a resistant measure of forecast accuracy (2013) by J. J. Montaño Moreno et al., published in the journal, Psicothema on p. 501.

The MAPE result can be interpreted as a stand-alone result, unlike the MAD or MSE results which would require multiple results to correctly assess accuracy.

A MAPE example is shown below. The MAPE result of 15.1% indicates that this is a good forecast based upon the table above. The internal auditor can attest that the forecast results from this mathematical programming planning model are accurate and in control.

Figure 12
Example of MAPE Results

		A	F	A-F	% A-F	| A-F |	A-F ^2	| (A-F)/A |
Month	**Year**	**Actual**	**Forecast**	**Diff**	**% Diff**	**Abs Diff**	**Sq of Diff**	**Abs of Error by Actual**
April	2016	49458	44000	5458	11%	5458.3	29792888.7	0.1104
May	2016	39905	46689	-6784	-17%	6784.3	46026936.2	0.1700
June	2016	41088	50127	-9040	-22%	9039.8	81718244.7	0.2200
July	2016	49708	52000	-2292	-5%	2292.1	5253888.9	0.0461
August	2016	40103	48124	-8020	-20%	8020.5	64328075.2	0.2000
September	2016	37886	44000	-6114	-16%	6113.7	37376907.1	0.1614
							MAPE	15.1%

Note. Adapted from Oracle® Fusion Cloud EPM Como Trabalhar com o Planning. Web in Oracle.

A non-parametric statistic that can be employed is the correlation coefficient (ρ) which will measure the correlation of the forecasts of the mathematical programming planning model and the actual results. The formula for the correlation coefficient is:

Figure 13
Formula for Correlation Coefficient

$$r = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}\sqrt{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}}$$

Note. Adapted from Correlation Coefficient with Minitab: Sample Correlation Coefficient (r) by Lean Sigma Corporation.

One would expect strong correlation between the actual results and the forecast if the mathematical programming planning model is accurate and its directives are adhered to by the user. A correlation coefficient of 95% or above would indicate that the model is accurate, but the entity may choose a different level specified by its policy.

A t-test of the correlation coefficient should be conducted if the variables are normally distributed. The null and alternate hypothesis of this t-test would be:
H0: $\rho = 0$
H1: $\rho \neq 0$
The actual results and the forecast should be normally distributed; otherwise, conclusions from the t-test could be misleading.

MS Excel analysis was conducted and the correlation coefficient from the inputs of the previous MAPE example is .25139, which indicates weak correlation between the actual results and the forecast and implies that the forecasts are fairly inaccurate.

Figure 14
Correlation Matrix

Actual	Forecast
49458	44000
39905	46689
41088	50127
49708	52000
40103	48124
37886	44000

	Actual	*Forecast*
Actual	1	
Forecast	0.25139	1

Note. Adapted from Oracle® Fusion Cloud EPM Como Trabalhar com o Planning. Web in Oracle.

MAPE and correlation analysis give conflicting results in this example and at this point the accuracy of the mathematical programming planning model is inconclusive. A fishbone diagram is a problem-solving approach that uses a fish-shaped diagram to model possible root causes of problems and troubleshoot possible solutions. A fishbone diagram should be employed to determine the inconsistent conclusions between MAPE and correlation analysis (Devor and Farnum, 2005).

Another popular non-parametric statistic is the χ^2 statistic. The χ^2 statistic is also called the "goodness-of-fit" statistic and its formula is:

Figure 15
Formula for the χ^2 statistic

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

O = the frequencies observed

E = the frequencies expected

$\sum$ = the 'sum of'

Note. Adapted from Statistical Methods – Chi-Square and 2x2 tables (2011) by Bean Around the World.

It compares two or more statistical data sets. It is used in data that consist of variables distributed across various categories and helps us to know whether that distribution is different from what one would expect by chance. The χ^2 statistic can determine the accuracy of a mathematical programming planning model by comparing actual results to the directives of a mathematical programming planning model.

Reliability of an internal audit can be improved by conducting the MAPE, correlation analysis and the χ^2 statistic together. Perform all three statistics, don't depend on a single statistical test. The ideal situation is if all three statistical tests come up with the same conclusion. The difficulty occurs when there is inconsistency in the conclusion between the statistical tests and this somehow must be resolved.

TIMING OF THE INTERNAL AUDIT

Degradation of forecast accuracy can occur over time so it is important to develop a schedule to consistently check accuracy. The schedule may be automated if the planning software allows this, which many do.

A hybrid schedule to assess forecast accuracy is recommended. A schedule of internal audits at regular intervals should be established. A regularly scheduled internal audit can become less intrusive and minimize the disruptions to the operations of the mathematical programming planning model.

The intervals between internal audits should be short if changes and updates are constantly made to the mathematical programming planning model. A short interval is necessary so that any adverse deviations can be controlled.

A major internal audit should be conducted annually per the *Sarbanes-Oxley Act of 2002*.

An internal audit should also be conducted if there are major changes and/or updates to the mathematical programming planning model. This would be analogous to the installation of a new software system. Beta testing of an updated algorithm should be planned and conducted. This would be important to check if the updated mathematical programming planning model is within specifications.

The timing of the internal audits of the mathematical programming planning model should be a hybrid schedule based upon the following recommendations:

- regularly scheduled internal audits throughout the year at set intervals
- a major internal audit annually per the *Sarbanes-Oxley Act of 2002*
- when a substantial change is made to the mathematical programming planning model

The internal auditor should make this a collaborative effort and communicate this to stakeholders of the mathematical programming planning model. Generally Accepted Auditing Standards should be strictly followed and all stakeholders should attest to the results of the various components of the internal audit.

CONCLUSION

The internal auditor should adhere to the general auditing plan guidelines of the Institute of Internal Auditors. This paper goes more in-depth and recommends additional and specified procedures for an audit plan of the internal audit of mathematical programming planning models. Here are the additions for the recommended audit plan, consistent with Generally Accepted Auditing Standards:

- An engagement with focuses on collaboration between all stakeholders
- Potential assistance of subject matter experts, mathematicians and operations researchers
- Legal compliance
    - *Sarbanes-Oxley Act of 2002*
- Audit of change management procedures
- Audit of compliance with directives of the mathematical programming planning models

- Descriptive statistics to assess mathematical programming planning model accuracy
  - mean absolute percentage error (MAPE)
  - correlation analysis
  - χ^2 statistic
- Timing of internal audits
  - regularly scheduled internal audits throughout the year at set intervals
  - a major internal audit annually
  - when a substantial change is made to the mathematical programming planning model

Mathematical programming planning models are an application of the decision sciences. Mathematical programming planning models are key contributors to the success of an enterprise because they can enhance operational efficiency and minimize operating costs. Internal auditing is a continuous improvement tool whose purpose is to enhance the performance of all processes. It will provide an overview of the effectiveness of processes and determine if a process should be corrected or improved upon.

Mathematical programming planning models and internal auditing are important to any business and the skilled internal auditor will combine these two functional areas to create a synergistic effect that is more than just the sum of its parts. This synergy will add economic value to any enterprise.

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