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It All Adds Up: Professional Development, Content Knowledge, and Self-Efficacy in Middle School Math Teachers

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LOYOLA MARYMOUNT UNIVERSITY

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Professional Development, Content Knowledge, and Self-Efficacy in Middle School Math Teachers

by

Wendy Creek

A dissertation presented to the Faculty of the School of Education,

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The math achievement of American students had been stagnant or falling since 2007, according to both national and international measures (NAEP, TIMSS, PISA). While the Common Core State Standards were partially a response to sinking levels of American math proficiency, those standards require a much greater depth of conceptual understanding of mathematics for teachers than previous standards, yet more than half of fifth- through eighth-grade math teachers are not certified to teach math. The federal government and school districts spend millions of dollars on teacher professional development, but little evidence shows what kind of professional development might be the most beneficial for math teachers. This study measured the impact of math content-based professional development on middle school math teachers. Findings suggest that the participating teachers' content knowledge about ratios and proportional reasoning

increased slightly during the study. Exit surveys indicated that the most recent PD session would have an impact on their teaching practice, although the impact would mostly be related to their pedagogy. However, there was little change in teachers' self-efficacy toward teaching mathematics. As research indicates, content knowledge in math is connected to student achievement, the implication of which is that middle school math teachers can increase their content knowledge through professional development. Given that access to higher-level math courses is critical to college success, and the foundation for those higher-level math courses begins in middle school, it is imperative that all students and their teachers are supported to be successful in math.

CHAPTER 1

INTRODUCTION

Background

American K–12 education is currently in a period of great flux. With fear of student achievement in the United States falling behind other countries, policy makers created, and the majority of states adopted, the Common Core State Standards (CCSS, corestandards.org) in English Language Arts and Mathematics between 2009 and 2013. These standards were originally launched by governors and state commissioners of education from 48 states, two territories, and the District of Columbia, and later championed by the federal government. These policy makers believed that standardizing what each state meant by "proficient" would help improve student achievement (corestandards.org).

The No Child Left Behind Act of 2001 was expected to increase student achievement, both nationally and internationally, and required standardized testing to determine whether content area objectives had been met. However, a look at the data reveal that the era of testing under NCLB may not have actually led to improved achievement. American fourth-graders currently rank 15th worldwide in mathematics achievement, and by the time they are in eighth grade have only managed to rank 9th (Trends in International Math and Science Studies, TIMSS, 2015). The latest results from the Program for International Student Assessment (PISA, 2015), indicate that American 15 year-olds rank 40th in the world mathematically. The most recent results of the National Assessment of Educational Progress (NAEP) also indicate that math scores for both fourth- and eighth-graders have dropped slightly since 2013 (NAEP, 2015). Overall, these scores indicate poor math achievement among American students.

In 2015, the Every Student Succeeds Act (ESSA) was passed, closing the legislative era of No Child Left Behind. ESSA allows states to set their own accountability goals within certain parameters, although schools are still required to test at least 95% of their students in reading and math in grades three through eight and once in high school, as well as separate their test data for certain subgroups of students in addition to the whole school (Education Week, n.d.). States must measure accountability based on five areas:

> achievement as measured by proficiency on annual state assessments, four-year cohort graduation rates for high schools, another academic indicator for elementary and middle schools (e.g., growth measure), progress in English language proficiency for English Learners (ELs), and at least one other indicator of school quality or student success that is valid, reliable, comparable, and statewide. (California Department of Education, 2017, p.2)

Although the measures of success are more varied than under NCLB, standardized assessments are still an important part of these new accountability measures.

Concern about student math performance requires educators to take a hard look at how students are taught. A recent study of California schools by WestEd indicates that while superintendents and principals believe the Common Core rollout has gone well, teachers do not (WestEd, 2016). One of the specific concerns cited by teachers was a lack of materials; although math textbooks were adopted by the state in 2013, none of them truly aligns to the Common Core Standards (Polikoff, 2015). The WestEd report indicated that teachers are primarily using on-line materials and teaching materials created by colleagues (WestEd, 2016). This is a major concern because it is estimated that more than half of fifth-through-eighth-grade teachers are not certified to teach math; most were also not math majors in college (Harris, Stevens, & Higgins, 2011). It is difficult to know if the teaching materials being used by teachers were created by mathematically proficient teachers.

In addition to teaching materials not being mathematically sufficient, another consideration related to math performance is that the Common Core mathematics standards require more advanced math concepts to be taught in earlier grades than previous standards. As such, the CCSS require teachers to have increased conceptual knowledge compared to previous sets of standards. However, most U.S. classrooms still focus on teaching mathematical procedures at the expense of teaching conceptual depth (Cobb & Jackson, 2011). In order to make the full shift to implementing the Common Core math standards, math teachers are going to need more math content knowledge to be effective.

Since 1989, when the National Council of Teachers of Mathematics (NCTM) released their *Curriculum and Evaluation Standards for School Mathematics*, they have been calling for teachers to rethink their math instruction, and to include a more constructivist approach. NCTM's 2006 *Curriculum Focal Points* form the basis for some of the Math Practice Standards that are a part of the Common Core State Standards in math from grades K–12 (corestandards.org). In the Common Core standards for math, rigor is specifically defined as a balance of conceptual understanding, application, and procedural fluency. Teachers using poorly aligned textbooks will not be balancing all three pieces of rigor because textbooks continue to emphasize procedures and rote memorization (Polikoff, 2015). A constructivist approach to teaching math allows students to “develop mathematical structures that are far more abstract, complex and powerful than the ones they currently possess so that they are increasingly capable of solving a wide variety of meaningful problems” (Clements & Battista, 1990, p. 7). However, teachers with lower self-efficacy are more likely to use lecture and text readings rather than a constructivist teaching approach (Czerniak & Schriver, 1994), possibly indicating a link between self-efficacy and a willingness to teach in a more student-directed or constructivist manner.

Based on the issues presented above related to student math achievement, with a specific focus on assisting teachers with more effective teaching methods, this study attempted to examine the impact of a mathematics content-based professional development on content knowledge for secondary math teachers who do not have full math credentials, on their self-efficacy related to teaching mathematics, and their willingness to adopt a more constructivist approach to teaching math. Accordingly, the purpose of this mixed methods study was to offer teachers content-based professional development in math concepts.

Problem Statement

With a focus on effective teaching practices to address concerns about poor math performance among American students, there has emerged a need for, and a great deal of research about, professional development (PD) for teachers. Unfortunately, a 2011 review by the U.S. Department of Education found that only nine out of 1,343 studies on teacher PD were based on research designed rigorously enough that the actual impact of the PD being studied could be verified. Of those nine studies, only four focused on the impact of PD on math achievement, but not one focused on middle school math (Garet et al., 2011). There is a clear need for research on teacher PD, specifically related to math and specifically related to middle school math.

Research on PD for in-service teachers has found that PD can impact teachers positively (Blank, 2013; Borko, 2004; Darling-Hammond, Chung Wei, Andree, Richardson & Orphanos, 2009; Foster, Toma, & Troske, 2013) but PD must be effective in order to change teaching practices; to create effective PDs it is crucial to first identify what effective teaching looks like (Cwikla, 2002). NCTM recommends that effective math teaching use a constructivist approach (National Council of Teachers of Mathematics, 1989). Given that math test scores for students in the U.S. are falling short when compared against international benchmarks, current teaching practices, relying on textbooks, are likely not effective.

The problem of low math achievement is further complicated by the fact that schools with high minority or low socioeconomic status (SES) populations are more likely to have teachers who lack full certification or who are teaching in a field other than the one they were originally certified to teach (Kohn, 2000; Peske & Haycock, 2006; Viruru, 2006). In fact, "nearly half the math classes in high poverty high schools and high minority high schools are taught by teachers who don't have a college major or minor in math or a math-related field, such as math education, physics or engineering" (Peske & Haycock, 2006, p. 3). Math teachers who were not able to meet the less rigorous expectations of previous sets of standards will not be able to effectively implement the Common Core math standards without support.

Given that districts may spend between 2% and 5% of their operating budget on professional development (Gulamhussein, 2013)—not including money provided by the states or federal government to support professional learning for teachers (Hill, 2009) —it is imperative that districts identify PD that has an impact on teaching practice. This study intended to show that teachers with better content knowledge may feel more empowered to teach math in more constructivist ways.

Purpose Statement

The purpose of this mixed-methods study was to examine an ongoing PD on mathematics pedagogy, administered through an entity known as CMAST, and highlight the impact of embedding content-based PD within it. Impact was measured by examining changes in teacher content knowledge, teacher self-efficacy toward teaching mathematics, and intended changes in teacher practice. Changes in teacher content knowledge and mathematics teaching self-efficacy were measured quantitatively from before to after the PD, while intended changes in practice were measured qualitatively after the PD.

Theoretical Framework

The National Research Council says that students must "construct their own mathematical understanding" (1989, p. 59). Sociocultural Theory (Vygotsky, 1978) posits that learning is a social process and that social interaction plays a fundamental role in the development of cognition. This constructivist view of learning is in line with Paulo Freire's (1970) critical theory, which suggests that students come with knowledge already inside them, and that knowledge should be built upon through discourse. The notion of students beginning with what they already know, and building understanding through discourse as a community is what reformers of math education have been hoping teachers would do for decades, and it is a way in which previously marginalized communities can gain an entry point into the culture of mathematics. By posing problems first, instead of frontloading vocabulary or relying on step by step textbook presentations, the nature of math as a process with which to solve problems becomes apparent.

Constructivism in mathematics teaching leads to students communicating about mathematics and being able to solve meaningful problems (Clements & Battista, 1990). Constructivism further encourages students by using ongoing assessment to measure progress rather than using a one-time test to determine student results (Ediger, 2001). Constructivism requires learners to self-organize in the context of a learning community (Cobb, 1994), thus making individual connections and connections throughout a sociocultural context. In order to truly make learning meaningful for and accessible to all learners, these ideas have to find their way into math classrooms.

Research has found that teachers with lower self-efficacy are less likely to use a constructivist approach; rather, they are more likely to use lecture and text readings (Czerniak & Schriver, 1994). The role of teacher beliefs and the impact of those beliefs on their teaching cannot be discounted. Bandura (1977) defined self-efficacy as "beliefs in one's capabilities to organize and execute the courses of action required to produce given attainments" (p. 3). There is some evidence that a teacher's beliefs about his or her ability to positively impact student learning has an effect on a teacher's actual success or failure in teaching (Althauser, 2015; Enochs, Smith, & Huinker, 2000; Henson, 2001; Swackhammer, Koellner, Basile, & Kimbrough, 2009). Some studies have shown that PD focused on how to teach content raised the self-efficacy levels of preservice teachers (Appleton, 1995; Palmer, 2001).

According to researchers (Telese, 2012; Wilson, Floden, & Ferrini-Mundy, 2001), an effective math teacher must have solid content knowledge. Because math content builds on foundational concepts, effective math teachers must also be fluent in the math it takes for students to learn the content for their grade level, the misconceptions students might have, and the general math required to teach a given topic, also known as Mathematical Knowledge for Teaching (MKT) (Hill, Rowen, & Ball, 2005). Teachers also need to understand how to blend content and pedagogy, which is known as Pedagogical Content Knowledge (PCK) (Shulman, 1987). The common link between all these types of knowledge is content knowledge.

Taken together, an important area yet to be investigated is the link between teacher content knowledge in math and teacher self-efficacy in teaching math among in-service teachers. Building on the logic that expert teachers are content experts, pedagogical experts, and have strong teaching self-efficacy, this study examined whether these variables ultimately led to teachers' willingness to step outside of a textbook and take a more constructivist approach to teaching math.

Research Questions

The following research questions guided this study.

1. What is the impact of a math content-based PD on the content knowledge of teachers who are teaching secondary math without a math credential?
2. What is the impact of a math content-based PD on the self-efficacy toward teaching math of teachers who are teaching secondary math without a math credential?
3. What is the impact of a math content-based PD on the intended classroom practices of teachers who are teaching secondary math without a math credential?

Middle school teachers, most of whom did not possess a math credential, participated in an ongoing PD administered through the Center for Math and Science Teaching (CMAST) at

Loyola Marymount University, Los Angeles, which focused on pedagogical techniques for teaching math. For the purpose of this study, these teachers also participated in two specific content-based PD sessions geared toward improving their mathematics content knowledge. These PDs included a component specifically focused on content knowledge in addition to typical pedagogical practices in math. These PD sessions were analyzed for this study. Three primary variables of interest (dependent variables) were measured to determine the efficacy of the content-based PD: (a) changes in middle school teachers' math content knowledge, (b) changes in their teaching self-efficacy in mathematics, and (c) teachers' intended classroom practices. It was anticipated that teachers who experienced the content-based PDs would improve their mathematics content knowledge, have increased self-efficacy in teaching math, and intend to include more constructivist pedagogy in their teaching practices.

Research Design and Methodology

A mixed-methods approach was used to answer the research questions for this study. Specifically, quantitative scores were calculated to assess teacher content knowledge before and after the PD, through a pre-and post-task (i.e., a math task that demonstrates content knowledge). Both the pre- and post-tasks were graded using the same rubric, created and vetted by content area experts. To assess teacher self-efficacy, the Mathematics Teaching Efficacy Belief Instrument (MTEBI), developed and validated by Enochs et al. (2000) was administered before the first content-based PD and after the second content-based PD. Finally, to assess the impact on teachers' intended classroom practices, an exit interview asked teachers to discuss their intended teaching practices, and to reflect on whether the PD would impact those practices.

This study used a purposeful sampling of teachers in a suburban public-school district in Southern California who were teaching secondary math (defined as grades six through 12), but did not all have a full math credential. A total of 22 teachers were initially selected for the study, although only 20 were able to fully participate in all phases of data collection.

The study used descriptive statistics to describe the characteristics of the participating teachers. A paired samples t-test indicated changes in the dependent variables from before to after the content-based PD.

Delimitations and Limitations

This study was delimited to middle school math teachers working in a public-school district in Southern California. Based on time constraints and research indicating that middle schools have a higher concentration of math teachers without math-specific credentials (Harris et al., 2011; Telese, 2012), the primary focus of this study was on middle school teachers. The sample size of teachers who were willing to participate in a professional development opportunity was limited, which subsequently limited the generalizability of the findings to the larger teacher population. As with any longitudinal study, attrition was a limitation. Additionally, the small sample size may not have yielded strong enough statistical power to determine meaningful effects. Finally, Kennedy (2016) noted a strong connection between teacher motivation to participate and PD impact, so the differing motivations of the teachers may impact the study and limit generalizability to teachers who are motivated to engage in PD.

Assumptions

This study assumed that the quantitative and qualitative data, including survey data, pre- and post-assessment data, and teacher exit interviews, reflected true and accurate information.

When possible, reliability of the instruments was assessed. The assessment task and rubric were vetted by content-area experts to increase the validity of the instruments to measure content knowledge. The survey items, borrowed from previous research (Enochs et al. 2000) were tested for internal reliability among the current sample.

Significance of the Study

This study is significant because it will directly impact in-service math teachers as well as offer insight about potential future training to preservice teachers. First, the study will benefit teachers who are currently teaching math but do not have a math credential by providing a PD on math content. Learning more content knowledge, delivered in a constructivist way, may present some guidance to these in-service teachers about how to change and strengthen their math teaching. The study may also benefit policy makers who may be considering changes to existing credentialing laws for secondary math teachers. As such, preservice teachers may be required to have greater content-knowledge in math prior to receiving a credential. School and district administrators are faced with decisions about where to spend limited professional development funds. This study may assist those decisions in that if the content-based PD is found to improve content knowledge and self-efficacy, then administrators may wish to spend their funding on such PDs. Lastly, the study has the potential to benefit students, who may reap the benefits of a constructivist approach to math teaching, should their teachers choose to adopt this approach.

Summary/Organization of the Study

Chapter 1 provided an overview of the study, including the purpose and parameters, and intendent significance. The chapter concludes with highlighting key terms and concepts. Chapter 2, the literature review, discusses five main topics: the culture of accountability and its effects on instruction, professional development for teachers, inquiry-based mathematics and the place of critical and sociocultural theory within inquiry-based mathematics, and convincing teachers to change their practice. Chapter 3 provides the study's methodology, which includes a description of the research design, sampling, instruments, procedures, and data collection. Chapter 4 will provide a detailed overview of the content-based PD created and implemented for the purpose of this study as well as situate the content PD within the larger context of the ongoing CMAST pedagogy PD. Chapter 5 will present the study's findings including the results of the pre- and post-PD self-efficacy survey and assessment task measuring content knowledge, as well as the results of the exit interviews. Chapter 6 will include a discussion of the results, a summary of the study, research conclusions, and implications for further consideration.

Definition of Terms

1. *Every Student Succeeds Act (ESSA):* A bill, signed into law on December 10, 2015, that both reauthorized and amended the Elementary and Secondary Education Act (ESEA) of 1965. The ESSA revised the earlier No Child Left Behind Act (NCLB) of 2001. The ESSA added new guidelines for standardized testing and put some of the control over that testing back in the hands of the states (Every Student Succeeds Act, n.d.).
2. *Local Control Action Plan (LCAP):* A part of the new Local Control Funding Formula introduced in California in 2013. It removed some of the stipulations on how schools and districts may spend money from the state, but requires that all expenditures are written into, and accounted for, through the LCAP.
3. *Local Control Funding Formula (LCFF):* A formula enacted in California in 2013–2014 that replaced the previous state system of education financing. "The LCFF establishes

base, supplemental and concentration grants in place of the myriad of previously existing K-12 funding streams" (CA Dept. of Education, n.d.). The LCFF was intended to give county and local education agencies more freedom in how funds are spent.

4. *Mathematical Knowledge for Teaching (MKT):* The math it takes for students to learn the content for their grade level, the misconceptions students might have, and the general math required to teach a given topic (Hill et al., 2005).
5. *National Assessment of Educational Progress (NAEP):*

> The National Assessment of Educational Progress (NAEP) is the largest nationally representative and continuing assessment of what America's students know and can do in various subject areas. Paper-and-pencil assessments are conducted periodically in mathematics, reading, science, writing, the arts, civics, economics, geography, U.S. history, and in Technology and Engineering Literacy (TEL). (NAEP - Overview, n.d.).

6. *No Child Left Behind (NCLB):* An upgrade to the Elementary and Secondary Education Act (ESEA) of 1965. It was passed by Congress in 2001 and signed into law by President Bush in 2002. The law required that all states have students designated "proficient" in national standardized tests by the 2013–2014 school year, and that schools be individually tracked by their progress toward that goal. That progress metric was known as Adequate Yearly Progress (AYP) and schools that failed to meet it ran the risk of being subject to an increasing list of sanctions beginning in the second year of failure (Klein, 2015).

7. *National Council of Teachers of Mathematics (NCTM):* A mathematics organization with 80,000 members in the United States and Canada. Their mission statement indicates that they are "the public voice of mathematics education, supporting teachers to ensure equitable mathematics learning of the highest quality for all students through vision, leadership, professional development, and research" (Overview - National Council of Teachers of Mathematics, n.d.).
8. *Pedagogical Content Knowledge (PCK):* "the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented and adapted to the diverse interests and abilities of learners and presented for instruction" (Shulman, 1987, p. 8).
9. *Program for International Student Assessment (PISA):*

    An international assessment of 15 year olds' reading, mathematics and science literacy that takes place every three years. PISA is coordinated by the Organization for Economic Cooperation and Development (OECD) and is administered in the Unites States by the National Center for Education Statistics. (NCES) (Program for International Student Assessment (PISA) - Overview, n.d.)

10. *Professional Development (PD):* An educational experience "related to an individual's work . . . people in a wide variety of professions and businesses participate in professional development to learn and apply new knowledge and skills that will improve their performance on the job" (Mizell, 2010).
11. *Self-Efficacy:* "beliefs in one's capabilities to organize and execute the courses of action required to produce given attainments" (Bandura, 1977, p. 3).

12. *Trends in International Mathematics and Science Study (TIMSS):*

> The Trends in International Mathematics and Science Study (TIMSS) provides reliable and timely data on the mathematics and science achievement of U.S. students compared to that of students in other countries. TIMSS data have been collected from students at grades 4 and 8 since 1995 every 4 years, generally . . . TIMSS is sponsored by the International Association for the Evaluation of Educational Achievement (IEA) and managed in the Unites States by the National Center for Education Statistics (NCES), part of the U.S. Department of Education. (Trends in International Mathematics and Science Study (TIMSS) - Overview, n.d.)

CHAPTER 2

LITERATURE REVIEW

The Culture of Accountability

In 1983, President Ronald Reagan's National Commission on Excellence in Education published *A Nation at Risk.* In the report, Terrel Bell, Reagan's Secretary of Education, claimed that the United States needed a competitive workforce, and that that need was not being met by the U.S. education system. In the words of the report, "if an unfriendly foreign power had attempted to impose on America the mediocre educational performance that exists today, we might well have viewed it as an act of war" (p. 1). While the 1993 Sandia Report attempted to refute the dire picture painted by *A Nation at Risk* (Huelskamp, 1993), subsequent researchers continued to dispute both sides of the issue, with each making claims as to how bad the state of U.S. education was, and the degree to which it needed to be reformed (Bracey, 1997; Stedman, 1994).

The legacy of *A Nation at Risk* continued into the new millennium, and culminated with the passage of the No Child Left Behind Act of 2001. This act was expected to deliver very impressive results, with 100% of U.S. students projected to be judged proficient in math and reading by 2014 based on standardized tests. The penalties for not achieving the required levels of growth in test scores each year were severe, and many politicians and education reformers applauded the beginning of an era of measurement, accountability, and quantifiable results. The previous 30 years in American education had been marked by a rise in public desire for regular testing and accountability (Loveless, 2006; Stedman, 1994). By 2001, there were 10 different tests in use in California alone for K–12 testing to determine achievement and college eligibility placement (California State Postsecondary Education Commission, 2001). However, as Loveless said in a 2006 report for the Brookings Institution on No Child Left Behind, “this observation, that Americans believe in test-based accountability and yet are leery of 'high stakes' testing, is important to note. Most people embrace accountability, but only an accountability devoid of unpleasant consequences” (p. 8).

The Era of Standardized Testing

The continued growth of standardized testing has given rise, however, to plenty of unpleasant consequences. As far back as 1990, Judah Schwartz was sounding a warning cry about standardized testing. In *The Intellectual Price of Secrecy in Mathematics Assessment*, she wrote:

> If we are going to use assessment to influence constructively the teaching and learning of mathematics, at least two conditions must prevail. The first condition is that the assessment instruments we use must not contradict, either explicitly or implicitly, our pedagogic goals. . . . The second condition that must be met . . . is that questions must be (at a minimum) mathematically interesting. (p. 55)

She also warned that having the tests shrouded in secrecy keeps our performance expectations secret, and that secrecy also dictates a “one correct answer” format—which is less interesting and less intellectually challenging—in addition to contradicting recent pedagogic goals set out in national standards (corestandards.org). Sixteen years later, Radhika Viruru (2006) wrote that “the air of secrecy that governs the tests is legendary: few people are allowed to see them and very few states make them available for public scrutiny” (p. 54). Viruru also pointed out that while they are public tests, they are written by private corporations over which the public has little, if any, control.

In addition to limiting interesting and intellectually challenging problems, standardized tests do a serious disservice to minorities and students from low socioeconomic backgrounds. In some districts, "the correlations between socioeconomic status and SAT-9 success have been reported to be as high as 0.9," suggesting that more affluent students score higher on the test (Boaler, 2003, p. 506). Minorities fare even worse, in part because standardized tests reflect the majority culture (Lomax, 1995; Viruru, 2006; Walden & Kritsonis, 2008). Norm-referenced tests compare students to each other on a normal distribution; thus any norm-referenced test will show half of the students scoring below the 50th percentile, because that is the median. In addition, test designers increase response variance by adding questions that rely on knowledge gained outside of school, primarily knowledge held by students with privileged backgrounds, as well as adding in questions specifically to test the reliability of the test. Those reliability questions are kept or discarded based on the number of students answering them correctly, and whether or not the students answering them correctly do well on the test as a whole. Based on the fact that minority students take tests in lower numbers than students from the majority culture, and the fact that minority students are less likely to do well on the tests overall, the reliability questions answered correctly by minority students are more likely to be discarded (Kohn, 2000). Progress targets set by the Federal Government are also unrealistic for the way in which students develop knowledge, especially in math. Ding and Navarro, in a 2004 longitudinal study of math achievement growth, found that "the assumption that children should show linear progress in norm-referenced tests in content areas is problematic and fails to account for the uneven way in which mastery of new concepts are achieved across developmental ages" (p. 249).

The passage of the Every Student Succeeds Act (ESSA) in 2015 brought about the official end of the NCLB Act, but in no way closed the era of standardized testing. While it grants more autonomy to states in determining when schools need improvement, ESSA by no means abolishes standardized tests. States are still required to use either a single annual summative assessment or several interim assessments throughout the year to produce a summative score. Additionally, while ESSA allows states to limit the aggregate time spent for assessments in each grade, states are still required to test students in grades three through eight and once in high school ("Every Student Succeeds Act," n.d.). California has chosen to integrate state and federal accountability systems into a California School Dashboard, which monitors not only current achievement on standardized tests, but also growth in test scores in addition to other performance measures (California Department of Education, 2017). Standardized tests have been reduced to a smaller measure of school success, but have not been eliminated altogether.

Standardized tests are developed in conditions of secrecy by corporations that make millions of dollars selling them to schools, while the public has almost no input into the process of test creation. They are geared— intentionally or not—toward reinforcing the ideals and achievement levels of the dominant culture. They also do not truly measure the way in which adolescents learn, yet government officials insist that they are necessary to determine whether effective instruction has taken place.

The Effects of Standardized Testing on Instruction

Studies have found that, as the emphasis on standardized testing has intensified, instruction has narrowed, with many teachers either resequencing topics to fit the emphasis of the test, eliminating topics not covered on tests (or covered in lesser amounts), or changing their teaching style to a more drill-oriented presentation (Blazer & Miami-Dade County Public Schools, 2011; Causey-Bush, 2005). All of these instructional issues are intensified in schools with high minority or low socioeconomic status (SES) populations, where teachers are more likely to lack full certification and districts are more likely to purchase scripted curriculum (Kohn, 2000; Viruru, 2006). Additionally, due to pressures associated with testing, large numbers of students are not finishing high school because, although they have finished coursework with passing grades, they are not able to pass an exit exam. The likelihood of dropping out before graduating increases for minority students, especially Black males (Nichols, Glass, Berliner, & Arizona State University, 2005; Walden & Kritsonis, 2008).

Defining Desired Results

> the rhetoric behind standardized testing has focused on the need for "measurability" and "accountability": vacuous terms in an educational context, terms that allow for new forms of domination.
>
> --Viruru, 2006, p. 53

The passage of the No Child Left Behind Act of 2001 was expected to increase student achievement, both nationally and internationally, and free students from discrimination by testing to determine whether or not objectives had been met, without defining what those objectives were in a uniform way. The development of the Common Core State Standards in

2009, was intended to define those objectives by developing a common set of standards for the nation. Governors and state commissioners of education from 48 states, two territories, and the District of Columbia spearheaded the initiative, believing that standardizing what each state meant by "proficient" would help improve student achievement (corestandards.org). By 2013, most states and U.S. territories had adopted the Common Core State Standards in English Language Arts and math. Based on the most recent Trends in International Math and Science Studies exam (TIMSS, 2015), American fourth-graders ranked 15th worldwide in mathematics achievement, and by the time they were in eighth grade had only managed to rank ninth. The latest results from the Program for International Student Assessment (PISA, 2015) indicated that American fifteen year olds ranked 40th in the world mathematically. The 2015 NAEP (National Assessment of Educational Progress) showed math scores for both fourth- and eighth-graders continuing to drop from 2013 levels (Nation's Report Card, 2015). At that point, those who were worried about U.S. competitiveness in mathematics education versus other nations would appear to be justified in their concerns. The Common Core standards are a step in the right direction, as it is imperative that the nation determines what is worth knowing and doing mathematically before it can come to a consensus on moving toward that goal (Cobb & Jackson, 2011; Huelskamp, 1993).

In 2013, the California state legislature allocated $1.25 billion from the general fund with the intent that the funds be used for implementation of the California Common Core State Standards, the Next Generation Science Standards, and the California English Development Standards (California Department of Education). The funds were allowed to be spent in three areas: professional development for teachers, administrators and paraprofessionals, instructional materials aligned to the aforementioned standards, and integration of the standards through technology. For the purposes of this study, the focus was on the professional development arena. Beyond the stipulation that funds can be spent on professional development, no other guidelines or requirements exist about what that professional development should look like, or must include. There is also no requirement for data collection to ascertain the effectiveness of said professional development. However, due to the large amount of money at stake, examining what the literature says about professional development for teachers is worthwhile.

Professional Development for Teachers

In 1999, Ball and Cohen summed up one of the problems with professional development for teachers thusly: “there is no coherent infrastructure for professional development. It is not the responsibility of any easily identifiable group or agency and so it happens everywhere-and hence lacks consistency, coherence, and curriculum” (p. 4). While there is a large volume of research on professional development for teachers, only a very small portion of the research has been designed with a sufficient level of rigor to verify the actual impact of the PD being studied. Of the 1,343 studies reviewed by the U.S. Department of Education, nine were deemed sufficiently rigorous; four of those were about the impact of professional development on math achievement; and none of them focused on middle school math (Garet et al., 2011). Some researchers claim that professional development has no impact on student achievement (Quint, 2011). Others claim that it is crucial to improving schools (Borko, 2004) and leads to student achievement gains (Blank, 2013; Darling-Hammond et al., 2009), while still others claim that before we can measure the effectiveness of professional development, we first need to define what effective teaching looks like so that teachers can set learning goals for their professional development (Cwikla, 2002).

Most districts do not have accurate records of how much money they spend on professional development, but some research suggests it is between 2% and 5% of their operating budget (Gulamhussein, 2013). Hill (2009) stated that estimations place that figure between 1% and 6%, but notes that this does not include money provided by the states or federal government to support professional learning for teachers. "For example, the National Science Foundation and U.S. Department of Education Math-Science Partnerships spent nearly $1.2 billion on mathematics and science learning for pre-service and in-service teachers between the years 2002 and 2007" (Hill, 2009, p. 473). In 2013, the State of California authorized $1.25 billion from the general fund for the purpose of "establishing high quality instructional programs for all pupils" (CA Dept. of Education, n.d.). While this money was intended to support the implementation of the Common Core State Standards in math and English, and Next Generation Science Standards, it has no stipulations other than establishing high quality instructional programs. The method of doing so was left up to individual school districts and, in California, must be spelled out in their Local Control Accountability Plan (LCAP). The LCAP is part of the new Local Control Funding Formula introduced in California in 2013. It removed some of the stipulations on how schools and districts may spend money from the state, but required that all expenditures are written into, and accounted for, through the LCAP. Nationally, the new ESSA puts stipulations on what professional development can look like, mentioning, among other requirements, that professional activities must be "sustained (not stand-alone, 1-day, or short term workshops), intensive, collaborative, job-embedded, data-driven and classroom focused…"

(“Every Student Succeeds Act,” n.d., p. 799). This proviso correlates with research on what characteristics of effective PD are, with effectiveness being defined as PD having an impact on either teacher practice or student outcomes (Dunst, Bruder, & Hamby, 2015; Kennedy, 2016). Given that one of the areas in which American students repeatedly fall short of international standards is math, there will likely be much math-focused professional development moving forward. But how does one even become a math teacher in the first place?

Becoming a Mathematics Teacher

The current study took place in California; as such, it was logical to examine the various paths to achieving a math credential in California. At the time of this study, one route to a teaching credential in mathematics in California was to complete an undergraduate mathematics degree, and upon completion of the undergraduate degree, complete a fifth year of study in a teacher preparation program. After successful completion of the teacher preparation program, which generally included a semester or two of student teaching, a preliminary teaching credential would be granted.

What differentiates elementary teachers from secondary teachers at this stage? Those who choose to teach elementary school in California must earn an undergraduate degree and demonstrate basic skills, which is achieved by either passing the California Basic Educational Skills Test (CBEST), passing the California Subject Examinations for Teachers (CSET) Multiple Subject plus Writing Skills Exam, passing a California State University Early Assessment or Placement Examination, or earning a high enough score on the SAT, ACT, or AP exam. Additionally, candidates must pass the (CSET) Multiple Subject plus Writing Skills Exam to demonstrate content knowledge. They must also pass the Reading Instruction Competency

Assessment (RICA). Those who choose to teach in a secondary school, defined in California as Middle or High School, traditionally complete an undergraduate degree in the subject in which they wish to teach. They may use the same pathways as elementary candidates for demonstrating basic skills, and may use an undergraduate degree from a Commission-approved university or a subject matter exam (CSET) to demonstrate subject matter competence. The teacher preparation portion of a traditional teacher credentialing path varies slightly for elementary and secondary candidates, but the core coursework around pedagogy is the same.

This is the traditional pathway; however, the California Commission on Teacher Credentialing website (http://www.ctc.ca.gov/credentials/teach.html) lists six different options for earning either an elementary or secondary credential, including coming through a university or district internship program or the Peace Corps. In addition, there are alternative credentialing pathways provided through organizations such as Teach for America, and it is possible to earn an undergraduate degree in a subject such as Business or Communications and use subject matter competency exams to earn a math credential. The CSET exam for math has three parts; algebra and number theory, geometry, probability and statistics, and calculus. Passing the first two subtests grants a Foundation Level Mathematics Credential, which allows single-subject math teaching up through geometry, which is generally ninth- or 10th-grade math in California. Passing all three subtests grants a full math credential and allows teaching through calculus.

There are many pathways to becoming a credentialed math teacher in California, and not all of them require an undergraduate degree in mathematics. Before determining if the lack of a degree makes a teacher more or less effective, borrowing an idea from Cwikla (2002) as mentioned earlier, it would be useful to define what effective math teaching looks like.

Defining Effective Math Teaching

What does it take to be an effective math teacher? Solid content knowledge is important, and may be of more importance than pedagogical knowledge (Telese, 2012), although it is difficult to measure actual teacher content knowledge and most studies rely on teachers' majors or coursework (Wilson et al., 2002). A recent study of fourth-grade teachers found that increasing teacher content knowledge did not impact student achievement (Garet et al., 2016), which may indicate the complexity of the interplay between content knowledge and pedagogy. One method of comparing teachers' pedagogical knowledge versus content knowledge may be to look at teachers certified through Teach for America (TFA). TFA recruits graduates from elite colleges who do not necessarily have a background in education and puts them into classrooms with minimal pedagogical training. Studies performed on (TFA) teachers indicate that when TFA teachers are compared to teachers in the same school with similar backgrounds and credentialing, they do slightly better in teaching math when it comes to test scores (Heilig & Jez, 2010; Raymond, Fletcher, & Luque, 2001); however, when it came to comparing TFA teachers to fully certificated teachers the TFA teachers did not fare as well, at least in the elementary grades (Darling-Hammond et al., 2005; Laczko-Kerr & Berliner, 2002). That TFA teachers would have more background in math and less in pedagogy would seem to indicate that, at least in math, and at least when dealing with under-certified teachers whose background in education is similar to a TFA teacher, content knowledge may be of more importance than pedagogical knowledge. Soine and Lumpe (2014) mentioned that teacher collaboration tied specifically to increasing their content knowledge in math had a larger impact on student achievement than teacher collaboration and reflection. This is of major concern, as it is estimated that more than half of fifth- through eighth-grade teachers were not certified to teach math, and were not math majors (Harris et al., 2011). These teachers will need content knowledge support, making Professional Development essential, but reaching teachers with low content knowledge can be difficult, as teachers with low content knowledge are unlikely to enroll in content-focused workshops (Hill, 2009).

In addition to content knowledge, math teachers must be fluent in the math it takes for students to learn the content for their grade level, the misconceptions students might have, and the general math required to teach a given topic, also known as Mathematical Knowledge for Teaching (MKT) (Hill & Ball, 2009). Linking the students to the content is what makes a teacher effective in any subject, since "an effective teacher attends to students' thinking and considers how best to link their current thinking with intended understandings of mathematical concepts, and uses this knowledge in pedagogical decisions" (Cwikla, 2002, p. 13). Common Core math increases the difficulty level both in content as well as in MKT, as it requires some more advanced concepts to be taught in earlier grades than previous standards, and necessitates more knowledge of how the standards build upon each other from grade to grade, as well as increased conceptual understanding compared to previous sets of standards. Currently most U.S. classrooms still focus on procedure at the expense of conceptual depth (Cobb & Jackson, 2011). In order to make the full shift to Common Core, all math teachers—even those with math degrees—are going to need more, and different, math knowledge than what they currently possess.

When it comes to teaching math specifically, how important are other types of knowledge that mix content and pedagogy? Lee Shulman (1987) described part of the knowledge base of teachers as Pedagogical Content Knowledge (PCK), defining it as "the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented and adapted to the diverse interests and abilities of learners and presented for instruction" (p. 8). While Shulman saw PCK as distinctly different from content knowledge and curricular knowledge, there is significant overlap between PCK and MKT as defined by Ball and others (Hill, Rowan, & Ball, 2005). Both PCK and MKT have been studied primarily in the elementary grade bands; and, while the relationship between PCK and content knowledge in secondary teachers was established in 2008, because secondary teachers need a higher level of math content knowledge than elementary teachers, their necessary knowledge may be more closely related to that of mathematicians (Matthews, 2013). The effects of PCK cannot be discounted, however, as university level math courses grant content knowledge, but do not explicitly deal with the interconnectedness of content and pedagogy (Goos, 2013).

This conundrum puts middle school math teachers in a unique position: they are considered secondary teachers for purposes of classification, but they are teaching students as they transition from an elementary understanding of mathematics into the beginnings of higher level math. Nancy Protheroe stated that instruction at the middle school level should "build on their emerging capabilities to think hypothetically, comprehend cause and effect, and reason in both concrete and abstract terms" (2007). This shift from concrete to abstract can be clearly seen in examining the Common Core math standards by grade level. The middle school standards begin in grade six with the end of fraction operations, but quickly develop into proportional reasoning, which sets the stage for understanding the constant of proportionality and slope in grades seven and eight. Students enter middle school finishing elementary concepts, and leave having begun algebra concepts. Middle school teachers need MKT and/or PCK in addition to content knowledge, but content knowledge itself is the foundation of both MKT and PCK. In California, sixth-grade teachers are currently not required to hold a single-subject credential; they may hold an elementary multiple-subject credential.

Desired Professional Development Outcomes

Given the multiple routes to becoming a mathematics teacher and the multiple types of knowledge teachers must possess to be considered effective, it is no wonder that Professional Development (PD) opportunities in teaching mathematics have become a common practice. However, defining what the outcomes of these PD opportunities should be varies greatly. Effective teachers "a) provide an equitable learning culture supportive of active collaboration, b) support the development of meaningful mathematical tasks, and c) provide ongoing feedback and formative assessment" (Cwikla, 2002, p.13). But a) and c), particularly, could be considered characteristics of any teacher leaving a teacher preparation program to begin teaching professionally. If the intent of professional development for in-service math teachers is to make them specifically better at teaching math, then why is there no literature documenting the specific outcomes all of this professional development is trying to achieve? While the goal of improving teachers and their teaching sounds promising, "improvement" is an ill-defined goal. Many professional development (PD) studies on math teaching have focused on implementation of a particular program or pedagogy, and measure whether teachers have implemented what they were specifically trained to do (Bell, Wilson, Higgins, & McCoach, 2010; Foster et al., 2013; Harris et al., 2011; Nipper et al, 2011). Others attempt to measure post-PD gains by looking at the impact on student test scores (Blank, 2013; Darling-Hammond et al., 2009; Quint, 2011;

Telese, 2012), however standardized test scores are not necessarily the best measurement of outcomes for all students as previously discussed.

Soine and Lumpe (2014) found that PD, which promoted "active learning" whereby teachers either observed other teachers or were themselves observed, planned lessons, and analyzed student work was effective in that it had some impact on teacher practice. Borko (2004) defined effective professional development as that which helps teachers "increase their content knowledge and change their instructional practices" (p. 5). Most professional development has not had lasting effects on changing teacher practice largely because most professional development does not work on teacher beliefs; in order for any large-scale change in math instruction to happen, teacher beliefs about math instruction must match those of the intended changes (Cobb & Jackson, 2011; Enochs et al., 2000). In order to understand how teachers' beliefs impact their instruction, one must examine the literature on professional development and teacher self-efficacy.

Self-Efficacy and Professional Development

Bandura (1977) defined self-efficacy as "beliefs in one's capabilities to organize and execute the courses of action required to produce given attainments" (p. 3). While teacher efficacy research suffers from poor construct validity issues, some evidence indicates that a teacher's beliefs about his or her ability to positively impact student learning have an effect on a teacher's actual success or failure in teaching (Althauser, 2015; Enochs et al., 2000; Henson, 2001; Swackhamer et al., 2009). Teachers who scored higher on self-efficacy measures were more likely to try new teaching ideas, be willing to grant more autonomy to students, be more attentive to lower-achieving students, and increase the self-efficacy of their students (Ross &

Bruce, 2007). Some studies show that professional development focused on how to teach content can raise the self-efficacy levels of preservice teachers (Appleton, 1995; Palmer, 2001). Because of the MKT component to teaching math, especially at the elementary level, studies about teacher self-efficacy in math tend to focus on professional development that combines some kind of content along with how to teach the content (Althauser, 2015; Ross & Bruce, 2007; Swackhamer et al., 2009). Other studies—not specifically about teacher efficacy, but about professional development for math teachers—that had a noted impact either on student achievement or student self-efficacy, which point out that lasting professional development must engage the teachers in a self-reflective component in addition to the content (Althauser, 2015; Dunst et al., 2015; Ross & Bruce, 2007; Stevens, Aguirre-Munoz, Harris, Higgins, & Liu, 2013). This is the critical connector between content knowledge, MKT and PCK; linking all three could have a large impact on teacher self-efficacy.

Toward More Inquiry-Based Mathematics

In turning on the television, surfing the Internet, or reading any current education blogs, hardly any time will pass before one hears an announcement about the importance of Science, Technology, Engineering and Math (STEM) education, and how vital hands-on learning is to the fostering of STEM excellence. STEM awareness has permeated American pop culture to the point where one of the highest-grossing movies of 2015, *The Martian*, was described by Variety as “a love letter to science” (“Ridley Scott’s ‘The Martian’ Is a ‘Love Letter to Science,’” n.d.). As interest in STEM education has risen, and 3D printers become more readily available, so too has interest in the *maker movement*. Public Radio International’s “The World” discussed the maker movement in 2014: a maker being defined as “anyone who’s creative and interested in technology, and they're interested in using the new technologies that are available to create devices and projects and products around them" ("Get ready for a manufacturing revolution, as the 'maker movement' goes mainstream," n.d.). Science and technology teachers are being encouraged to make classes more hands-on, and to think outside the textbook. Salman Khan, the founder of the Khan Academy website, promotes an hour of code each year that encourages kids and adults alike to learn the basics of computer programming. Unspoken in all the furor over hands-on STEM is that the M in STEM has a long way to go before it is ready to support the kind of experiential learning that science and technology are moving toward. This is problematic, because, as the adage goes, "you can have math without science, but you can't have science without math." Indeed, *The Martian* can be seen as much a love letter to math as it is to science, as the main character is forced to repeatedly apply math through problem-solving, calculating and recalculating furiously as his fortunes fluctuate. There is plenty of math hand-in-hand with the science and technology. How can teachers access a more inquiry-based approach in math? And, why should they?

Math teachers primarily use math textbooks to teach math, as they were taught in their teacher preparation programs (Cobb & Jackson, 2011; Goos, 1999; Telese, 2012). The problem with this is that most textbooks are poorly aligned to the Common Core standards, and are still emphasizing procedures and rote memorization to a much greater degree than the standards themselves (Polikoff, 2015). Rigor is defined in the Common Core standards for math specifically as a balance of conceptual understanding, application, and procedural fluency (corestandards.org). This understanding means that any teacher depending primarily upon the textbook to plan instruction is going to be focusing only on the procedural third of instruction.

To balance instruction on all three components of rigor, teachers will have to supplement their textbooks, and in fact a large percentage of teachers are supplementing with materials created by either themselves or their colleagues (Kane, Owens, Marinell, Thal, & Staiger, 2016; WestEd, 2016), leading to concerns over whether teachers have sufficient time, guidance, or content expertise to effectively put together a coherent curriculum out of disjointed pieces (NCTM, 2016; Zubrzycki, 2016). Additionally, even if curricula contain problems meant to develop understanding of a concept, U.S. teachers typically turn conceptual problems into procedural problems by either providing students with formulas or shortcuts from the beginning, or giving students answers before they have an opportunity to explore the problem themselves (Stigler & Hiebert, 2004).

Clements and Battista (1990) stated that the two major goals of taking a constructivist approach to teaching mathematics are as follows:

> First, students should develop mathematical structures that are far more abstract, complex and powerful than the ones they currently possess so that they are increasingly capable of solving a wide variety of meaningful problems. Second, students should become autonomous and self-motivated in their mathematical activity. Such students believe that mathematics is a way of thinking about problems . . . They see their responsibility in the mathematics classroom not so much as completing assigned tasks, but as making sense of, and communicating about, mathematics. (p. 7)

Taking a constructivist approach to mathematics instruction can allow students to access all three components of rigor.

Critical and Sociocultural Theory in the Mathematics Classroom

Mathematics, perhaps more than any other subject taught in school, suffers from a phenomenon whereby, at least in the United States, it is culturally acceptable to be considered "bad at math" in a way that would not be considered acceptable in other subjects. John Allen Paulos terms this "innumeracy" and discusses it in detail in his 1988 book *Innumeracy: Mathematical Illiteracy and its Consequences.* He mentioned that one of the ill effects of innumeracy is a tendency to misunderstand science and believe in pseudoscience. Given that so much public policy is driven by science and technology, members of our society must understand what they are seeing and know when numbers are being manipulated. Marilyn Frankenstein (1983), writing about applying critical pedagogy to mathematics said:

> Knowledge of basic mathematics and statistics is an important part of gaining real, popular, democratic control over the economic, political and social structures of our society. Liberatory social change requires an understanding of the technical knowledge that is too often used to obscure economic and social realities. When we develop specific strategies for an emancipatory education, it is vital that we include such mathematical literacy. (p. 315)

Mathematics education, then, must be for everyone, and must have an entry point for everyone. Currently that is not the case, as the math achievement among low-SES and non-White students lags behind that of White and high-SES students ("NAEP - 2015 Mathematics & Reading - Mathematics - National Scores by Student Group," n.d.).

Nearly 30 years ago the National Council of Teachers of Mathematics (NCTM) called for math empowerment for students who had been denied academic success in math. In 1989, the

NCTM released its *Curriculum and Evaluation Standards for School Mathematics.* These standards called for a much more constructivist approach to teaching mathematics, and asked for teachers to rethink their approach to teaching. Additionally, they called for equity and mathematical power for females and students of color. Those standards received harsh criticism due to their emphasis on conceptual understanding over rote memorization and, in 2000, NCTM released the *Principles and Standards for School Mathematics,* which balanced constructivism and computation more evenly. In 2006, NCTM released its *Curriculum Focal Points*, some of which form the basis for the Math Practice Standards that are a part of Common Core State Standards in math from grades K–12.

If math empowerment for women and students of color can be accomplished with a more constructivist approach, as called for by NCTM, and the roots of that reform inform the current Common Core Standards, then now is an excellent time to rethink classroom practice to be more equitable for those students. The task will not be easy, as most teachers default to teaching how they were taught (Watson, 1995), but with so many federal dollars available for professional development there may be no better funded time. According to The National Research Council (1989):

> In reality, no one can teach mathematics. Effective teachers are those who can stimulate students to learn mathematics. Educational research offers compelling evidence that students can learn mathematics well only when they construct their own mathematical understanding…This happens most readily when students work in groups, engage in discussion. (p. 59)

If the recommendation for teachers is that they should facilitate student discussion to help students construct their own mathematical understanding, then a sociocultural perspective would be useful. Cobb (1994) made the case that learning math is both about constructing individual meaning, and “enculturation into the mathematical practices of wider society” (p.13). Because sociocultural theory takes into account the learner’s environment and surroundings, and it is impossible to separate students from their environment, Cobb’s is a case worth considering.

Sociocultural Theory and Mathematics

Sociocultural Theory (Vygotsky, 1978) posits that learning is a social process and that social interaction plays a fundamental role in the development of cognition. The Common Core Math Standards include eight Standards for Mathematical Practice, which are intended to help students develop “mathematical habits of mind” (corestandards.org). Developing those habits means to learn to think and approach problems like a mathematician, which requires enculturation into the thought processes of the mathematics community. Sociocultural theory can bridge the divide between the community of the students and the community of mathematicians by helping students construct meaning as a classroom community, then guiding that community toward the mental practices of the math community. As an example of how learning is tied to social and cultural influences, Vygotsky described a zone of proximal development (ZPD) as a place between a student’s current and potential understanding. Only in collaboration with a “more knowledgeable other” will a student stretch to move into and past the ZPD. Within their community of math learners, students can take definitions, procedures, and algorithms and use discourse to construct their own meaning, connecting it to their personal experience (Steele, 2001). Additionally, allowing students to construct discourse in the classroom empowers the students, since the choices that the teacher makes in building classroom discourse “constitute power relations in the mathematics classroom” (de Freitas et al., 2012). Accepting that students come into a classroom with some level of knowledge, and being willing to shift power relations in the classroom are some of the tenets of Critical Theory and Constructivism.

Critical Theory and Constructivism

Paulo Freire (1970) called for a change in the teacher-student relationship. In traditional schooling, teachers saw themselves as the subjects, and students merely as objects, or as empty vessels to be filled. Freire’s critical theory posits that students come with knowledge already inside them, and that knowledge should be built upon through discourse. This notion is what reformers of math education have been hoping teachers would do for decades, and is a way that previously marginalized communities can gain entry into the culture of mathematics. By posing problems first instead of frontloading vocabulary or relying on step-by-step textbook presentations, the nature of math as a process with which to solve problems becomes apparent. Allowing students to help determine what problems are worth solving empowers them to use math as a tool for their own liberation. Frankenstein (1983) said that “knowledge does not exist apart from how and why it is used, in whose interest” (p. 316). Students need to know who is using math and for what purpose.

Fosnot (2005) highlighted the goal of constructivism as “autonomy, mutual reciprocity of social relations, and empowerment…” (p. ii). Far from believing that learners come without any knowledge, and must be handed all knowledge by their teacher, constructivism asks teachers to plan experiences in which students can look for patterns, raise questions, make models and create their own knowledge with the guidance of the teacher (Cobb, 1994; Fosnot, 2005; Watson, 1995). In order for mathematical ideas to be meaningful to students, they must be connected to students' existing knowledge by having the student reflect on what they are doing (Clements & Battista, 1990). Constructivists look to Piaget and Vygotsky as sources for how to help teachers guide students, but their theory also aligns with Freire's beliefs about the knowledge that pre-exists in students, and how discourse leads to building on that knowledge.

Those who are opposed to constructivism in mathematics education claim that it is because math is not a natural discipline, but was invented by humans. While this is true, it is equally true that from very early ages, children sort their toys and objects around them into categories based on attributes, or quantities. This is the foundation for algebra. Students cannot be expected to learn everything they will need to know; there will need to be guidance and clarification from the teacher. Marilyn Burns (2014) differentiated between knowledge that can be found by logic, versus knowledge that is "part of agreed on social conventions" (p. 65). As an example, she discussed guiding students through an investigation about Pi. While the students were able to measure the circumference and diameter of many differently sized circles and see that the relationship between the two was always the same, the teacher will need to tell the students that this relationship is represented by the number Pi.

Convincing Teachers to Make the Change

Given that there have been unsuccessful attempts at math reforms before, the problem is how to provide professional development to teachers that will convince them to change their practice—and to do so long term. Each school culture will need to change, as teachers within a culture tend to teach in similar ways (Stigler & Hiebert, 2004). Utilizing constructivism in mathematics, especially through a critical and sociocultural lens, will require everyone in education to reexamine their beliefs and be open to learning new ways to engage in the old business of education (NCTM 1989; NRC 1990; Watson, 1995). In order to permanently change teacher practice, professional development must involve a self-reflective component (Bruce & Ross, 2008; Dunst et al., 2015). Only by reflecting on their learning throughout the process can teachers see themselves as both learners and future practitioners. Since this requires teachers to create different types of experiences for their students, they themselves are going to have to experience what their students will be expected to experience (Watson, 1995). Additionally, teachers will need to add to their existing content knowledge before using that knowledge to build MKT or PCK. This model will build teacher self-efficacy, and empower teachers to explore math outside the textbook.

Guiding students to discover how to use math as a tool to solve problems in their own lives can give all students a personal connection to the content, which should be the goal of any teacher. Having a personal connection to math will help students become interested in future careers in STEM fields, which are currently suffering from a lack of diverse perspectives (Tsui, 2007). The problems of the future will affect all students, not just male students or White students. All students should be involved in solving them.

Effective math teaching depends upon teachers who have the knowledge of their content, the confidence to use that knowledge, and the understanding of how to use that knowledge to guide students in creating and organizing new knowledge for themselves. The goal of this study was to help teachers feel empowered to support their students. The following chapter will discuss how the study unfolded.

CHAPTER 3

METHODOLOGY

American 15 year olds were ranked 40th in the world mathematically based on their scores on the most recent Program for International Student Assessment (PISA) exam. Given the current focus on Science, Technology, Engineering and Math (STEM) education, American students have a long way to go to strengthen the “M” in STEM. The recent passage of the Every Student Succeeds Act (ESSA) continues to require annual testing for students in grades three through eight, and requires at least one test in high school, indicating that the United States is not yet finished with standardized testing. Renewing a commitment to standardized testing also renews the commitment to the notion of accountability, which is also mentioned in ESSA. While the responsibility for identifying schools in need of support has been shifted to the states, ESSA still requires the formation of a statewide accountability system (“Every Student Succeeds Act,” n.d.). That accountability system is unlikely to show American students becoming more competitive internationally unless something about the way math is taught in American schools changes.

This mixed-methods study intended to help schools and school districts by examining how to support teachers who were currently teaching middle school math without a math credential. Specifically, this study examined the effects of a math content-based professional development offered to middle school teachers, and measured the impact of the professional development on their content knowledge and their self-efficacy toward teaching math. The study also attempted to measure whether exposure to content knowledge impacted teachers’ willingness to adopt a more constructivist approach to teaching math.

Research Questions

It is estimated that more than half of fifth- through eighth grade teachers do not have a math credential and lack sufficient background knowledge to successfully teach math (Harris et al.,2011). The recently adopted Common Core State Standards in mathematics require higher cognitive demand than previous sets of standards. Cognitive demand increases when students are asked to make conjectures, construct proofs, make connections, and solve nonroutine problems (Porter, McMaken, Hwang, & Yang, 2011). This requires teachers to be able to support students in ways that are not explicitly spelled out in textbooks, and ways that the teachers themselves were not taught math. This means that schools and school districts are going to need to offer some type of content support to their in-service math teachers, especially since some research has tied low teacher self-efficacy to a more lecture-based, teacher-directed classroom approach (Czerniak & Schriver, 1994). If teachers rely on lecturing as their pedagogical technique because they lack conceptual knowledge, then support in content knowledge is necessary and may improve teaching math self-efficacy and lead to a willingness to teach in a more student-directed and constructivist manner.

The following research questions guided this inquiry into the connection between teacher content knowledge in math, teacher self-efficacy in teaching math, and teacher willingness to teach in a more student-directed, less teacher-directed manner.

1. What is the impact of a math content-based PD on the content knowledge of teachers who are teaching secondary math without a math credential?
2. What is the impact of a math content-based PD on the self-efficacy toward teaching math of teachers who are teaching secondary math without a math credential?

3. What is the impact of a math content-based PD on the intended classroom practices of teachers who are teaching secondary math without a math credential?

For the purposes of this study, secondary math is defined to be grade six (middle school) or higher; teachers without a math credential are considered those who have a multiple subject elementary credential, or no single-subject math credential, but are teaching secondary math.

Methods

This study utilized quantitative and qualitative data to answer the research questions. The results of this mixed-methods study could help inform schools and school districts in how to support their in-service math teachers in connecting math content to students. A mixed-methods research approach was used to answer the research questions measuring three dependent variables: math content knowledge; teaching math self-efficacy; and willingness to include constructivist pedagogy in their math classrooms.

To assess the impact on teacher content knowledge before and after the PD, teachers were given a pre- and post-task. These tasks covered the Common Core content standard–related to ratios and proportions, which is typically taught in grades six and seven. These tasks were scored by a pair of experienced math teachers with math credentials who were not involved in the study.

To assess teacher self-efficacy toward teaching math, participants were given the Math Teacher Efficacy Beliefs Inventory (MTEBI) before the first PD and after the second PD. The MTEBI has strong reliability, and factorial validity was established by traditional factor analysis, as well as a more rigorous confirmatory factor analysis (Enochs et al., 2000). This instrument has been used to measure teacher self-efficacy in math and permission was granted to use this instrument in the current study.

Finally, to assess the impact of the PD on teachers' intended classroom practices, teachers were given the opportunity to record an exit interview, which included questions designed to measure their intended pedagogical practice with regard to what they had learned in the PD sessions. Questions in the exit interview were designed to assess the impact of the PD on the teachers and on their teaching practice.

Participants

Participants were recruited from middle schools in a public-school district in Southern California. The participants were teachers with whom the researcher had had a professional working relationship for the previous two years; as such, the study involved purposeful convenience sampling techniques. Twenty teachers participated. Participants were teachers who were credentialed to teach middle school math, either through holding a multiple-subject credential and teaching sixth-grade math, or holding a math credential or supplements to the multiple subject credential and teaching seventh-grade math, eighth- grade math, or Algebra 1. Participants were selected from a district with whom the researcher had a working relationship, as that increased the likelihood of participants finishing the entire study.

Procedures

Recruitment. After obtaining approval from the University Institutional Review Board (IRB), the researcher obtained permission from the respective school district to contact teachers about the study. Participants were contacted by email, informed of the topic and purpose of the study, assured their participation was voluntary, and assured of confidentiality.

Pre-assessment. Participants were given the MTEBI electronically and asked to complete it before the first PD. The MTEBI measured teacher self-efficacy and required them to reflect on their beliefs about teaching mathematics. Participant demographics such as gender, number of years teaching mathematics, and type of credential held were also collected. The pre-task, to measure their baseline levels of content knowledge, was administered at the start of the first PD and asked them to engage in a problem requiring knowledge of ratios and proportions.

Professional development. The content-based professional development included two sessions lasting two hours each. The content of the first half of each PD focused on math content related to ratios and proportions, and the second half of the PD was about inquiry-based pedagogy. The content portion of the PD unfolded over four phases. Phase one began with teachers being led through an activity designed to increase their understanding of ratios and proportions in a constructivist manner, so that they could experience what their students were expected to experience (Watson, 1995). For example, the teachers began with a question that required them to clarify their understanding of the idea of ratios and proportions and be supported by the facilitator through questions designed to refine their understanding, rather than being given lecture notes on the topic and guided through practice problems (See Chapter 4 for PD Lesson Plan).

Phase two situated the content that teachers were required to teach their own students vertically within the math progressions. In other words, teachers were exposed to the content that precedes ratios and proportions, then the actual content, and finally, the content that follows ratios and proportions. One of the major shifts in Common Core mathematics is the idea of coherence, which is understanding how content connects within and across grades (corestandards.org). Connecting content within a context helps students make neural connections and strengthen long-term memory, which increases the likelihood that students will be able to access information when they need it (Souza, 2007).

Phase three asked teachers to reflect on themselves as learners and asked how they can link their own learning to their classroom practice. Bruce and Ross (2008) stated that in order to permanently change teacher practice, professional development must involve a self-reflective component, and that teachers must reflect on their learning to see themselves both as learners and future practitioners. Finally, phase four asked teachers to use their knowledge of both the content and its vertical alignment to plan a lesson experience for their students.

Post-assessment. Participants were given the post-task after completing the second PD to measure changes in content knowledge. The post-task asked them to engage in a problem very similar to the pretask, requiring knowledge of ratios and proportions. After completing the post-task, participants were asked to sit for an exit interview and describe verbally whether they believed the PD would impact their future teaching and, if so, how. Finally, the day after the second PD, participants were sent the MTEBI and asked to take it again as a postsurvey. The MTEBI required them to reflect on their beliefs about teaching mathematics and measured self-efficacy.

Instruments

Quantitative data collection. Two different sources of quantitative data were gathered to analyze the impact of the PD on teacher content knowledge and teacher self-efficacy toward teaching math.

Pre- and Postcontent Tasks. To determine the impact of the PD on teacher content knowledge, teachers were asked to complete a task based on the Common Core Math Standard 7.RP.2 (recognizing and representing proportional relationships between quantities) from Inside Mathematics, both before the first PD and after the second PD. Tasks from Inside Mathematics were chosen from their task library, which is aligned to Common Core Math Standards, and includes rubrics for scoring. For example, teachers were asked to look at a photograph that had been resized from an original, and assess what the dimensions of the new photograph were when some of the measurement information was missing. Performance on both tasks was assessed by two credentialed math teachers using a rubric developed by Inside Mathematics (http://www.insidemathematics.org/performance-assessment-tasks). Inside Mathematics is an outgrowth of the Silicon Valley Mathematics Initiative, a group funded by the Noyce Foundation for the expressed purpose of helping teachers provide the best math instruction they can. Inter-rater reliability was established by training the scorers to use the task rubric consistently. To confirm inter-rater reliability, the researcher checked for consistency after the scorers assessed the tasks.

MTEBI. To determine the impact of the PD on teacher self-efficacy toward teaching mathematics, teachers were asked to complete the MTEBI before the first PD and after the second PD. The MTEBI had 21 items, eight on math teaching outcome expectancies, and 13 about personal math teaching efficacy. It asked teachers to rate how strongly they agreed with statements such as: “I know how to teach mathematics concepts effectively”; “Given a choice, I will not invite the principal to evaluate my mathematics teaching”; “When teaching mathematics, I will usually welcome student questions.” (See Tables 1 and 2).

Table 1

MTEBI Statements Measuring Outcome Expectancy

1. When a student does better than usual in math, it is often because the teacher exerted a little extra effort.

2. When the math grades of students improve, it is often due to their teacher having found a more effective teaching approach.

3. If students are underachieving in math, it is most likely due to ineffective math teaching.

4. The inadequacy of students' math background can be overcome by good teaching.

5. When a low-achieving child progresses in math, it is usually due to extra attention given by the teacher.

6. The teacher is generally responsible for the achievement of students in mathematics.

7. Student achievement in math is directly related to their teacher's effectiveness in math teaching.

8. If parents comment that their child is showing more interest in math at school, it is probably due to the performance of the teacher.

Overall Cronbach's Alpha = .77

Note. Expectancy items were followed with a 5-point Likert Scale with response options ranging from 1 = *Strongly Agree* to 5 = *Strongly Disagree*.

As seen in Table 1 above, these eight survey items measured teachers' Outcome Expectancy for students based on how they felt the influence of a teacher in general impacted student math learning outcomes. Table 2 below cites the 13 items that measured Personal Math Teaching Efficacy Belief.

Table 2

MTEBI Statements Measuring Personal Math Teaching Efficacy Belief

1. I will continually find better ways to teach math.
2. Even if I try very hard, I will not teach math as well as I will most subjects.*
3. I know how to teach math concepts effectively.
4. I will not be very effective in monitoring math activities.*
5. I will generally teach math ineffectively.*
6. I understand math concepts well enough to be effective in teaching math.
7. I will find it difficult to use manipulatives to explain to students why math works.*
8. I will typically be able to answer students' questions.
9. I wonder if I will have the necessary skills to teach mathematics.*
10. Given a choice, I will not invite the principal to evaluate my math teaching. *
11. When a student has difficulty understanding a math concept, I will usually be at a loss as to how to help the student understand it better.*
12. When teaching math, I will usually welcome student questions.
13. I do not know what to do to turn students on to math.*

Overall Cronbach's Alpha = .74

Note. Expectancy items were followed with a 5-point Likert Scale with response options ranging from 1 = *Strongly Agree* to 5 = *Strongly Disagree*. Items marked * were reverse-coded during analysis.

Table 2, above, shows the items that measured Personal Math Teaching Efficacy Belief, which reflects how teachers feel they personally impact student math learning outcomes.

Qualitative data collection. To provide the voice of the teachers in studying their self-efficacy, qualitative data were also collected.

Interviews. To determine the impact of the PD on the teaching practice of the participants, exit interviews were conducted. Participants were given 10 minutes to reflect on interview questions, then they were given an iPad and asked to record their answers to interview questions such as: "How likely is it that today's PD will impact your classroom practice?" and "Describe one takeaway from today's PD that you found helpful." Specifically, teachers were asked if they foresaw changes in their teaching techniques, and their responses were analyzed to determine whether those changes aligned to a more constructivist approach. Participants were asked to use the video recording feature on the iPads with the iPad pointed toward the ceiling to preserve some measure of anonymity in order to allow participants to feel more freedom in sharing their thoughts, although three of the participants did video their faces.

Analytical Plan

This study used descriptive statistics to describe the credential levels of the participants, as well as the number of years they had been teaching math. A dependent samples t-Test was run to determine changes in the mean scores of the dependent variables of content knowledge and self-efficacy from before to after the PD. Teachers' intentions to implement constructivist approaches were measured via the exit interview. Exit interviews were also coded for themes related to pedagogical practice, especially related to constructivist techniques or changes indicating a student-centered classroom.

Delimitations and Limitations

The small sample size of teachers and choosing teachers from public schools in Southern California limited the generalizability of findings to other geographical regions and nonpublic schools. This study did not address teachers with limited math credentials or supplementary authorizations who were teaching secondary math at the high school level, so the findings might not apply to those teacher populations. Further, these teachers were selected purposefully and based on pre-existing relationships with the researcher, limiting generalizability to teachers who may have had no interest in participating in PDs. The study did not intend to establish causality. However, the pre-post design afforded a sense of the PD's impact on the three dependent variables. Additionally, the study was performed in a very limited period of time with limited ability to follow up with teachers; future studies would benefit from an analysis over a longer period of time and with more exposure to content training.

CHAPTER 4

THE CMAST PROGRAM

Professional development (PD) for teachers is a multimillion-dollar industry. School districts spend as much as 5–6% of their operating budget on PD (Gulamhussein, 2013; Hill, 2009), with additional money provided by the state and federal governments specifically for teacher training. There is no regulatory agency for teacher PD, and there is very little research done with enough rigor to determine whether the PD had any impact (Garet et al., 2011). With the adoption of the Common Core State Standards in 2013, teachers have had to look at content with a fresh eye. Math teachers in particular are being asked to teach math by developing conceptual understanding in students in a way that differs greatly from previous sets of standards (corestandards.org). Given that middle school teachers teach secondary math, but most of them do so without full math credentials (Harris et al., 2011), there is a great need for content knowledge support for middle school math teachers. This study examined the effect of a content-based PD on mathematics content knowledge, self-efficacy toward teaching math, and intended teaching practices among a group of middle school math teachers.

The new Every Student Succeeds Act (ESSA), stipulates that PD for teachers must be "sustained (not stand-alone, 1-day, or short-term workshops), intensive, collaborative, job-embedded, data-driven and classroom focused" ("Every Student Succeeds Act," n.d., p. 799). At the time of this study, the Center for Math and Science Teaching (CMAST) at Loyola Marymount University (LMU) was offering such PD to math and science teachers throughout Southern California. This chapter will begin with an overview of the University and the School of Education in which CMAST resided, will describe what the CMAST program was, and then describes the content-based PD developed for the purpose of this research study.

Background

Loyola Marymount University

Loyola Marymount University (LMU), founded in 1911, is a private Catholic University, steeped in the Jesuit and Marymount traditions, and sponsored by the Society of Jesus, the Religious of the Sacred Heart of Mary, and the Sisters of St. Joseph of Orange. Located in West Los Angeles. LMU's mission is "The encouragement of learning, the education of the whole person, the service of faith and the promotion of justice" (http://www.lmu.edu/about/factsfigures/). As of Fall 2016, LMU had 6,162 undergraduate and 2,099 graduate students enrolled, with approximately 24% of the undergraduates being from out of state, and 8.5% from out of the country. The campus is ethnically diverse: according to the school's website, for the 2016–2017 school year, African American students were 6.3% of enrollment, Asian students were 10.6%, Hispanic/Latino students were 21.2%, White/Non-Hispanic students were 44.9%, and multiracial students were 8.03%.

The LMU School of Education (SOE). As of Fall 2016, the LMU School of Education enrolled approximately 250 undergraduate students, and 1,500 graduate students. The SOE offered 25 degree, credential, or certificate programs, 17 of which had a graduate-level option and one doctoral option. In addition to the university mission, the School of Education mission is expressed as Respect, Educate, Advocate, and Lead (REAL) and its purpose is "the encouragement of life-long learning, academic excellence, the education of the whole person, and the promotion of service and justice for all" (http://soe.lmu.edu/about/mission/). The School of Education operates under a Conceptual framework that “focuses on educational success for all learners through respect, education, advocacy, and leadership” (http://soe.lmu.edu/about/mission/conceptualframework/). Many SOE alumni live and work in the greater Los Angeles area, and various community organizations partner with the SOE, expanding its impact throughout one of the most densely populated regions of the United States (http://soe.lmu.edu/about). It was in this environment that the Center for Math and Science Teaching (CMAST), and Mathematics Learning by Design (MLD) were created.

The Center for Math and Science Teaching (CMAST)

The Center for Math and Science Teaching (CMAST) resides within the SOE at LMU. This center serves to educate in-service math and science teachers in inquiry-based pedagogy. The Center is staffed by six faculty members, one dedicated to science and five dedicated to math, who provide professional development (PD) and coaching support in K–12 schools throughout the greater Los Angeles area. Faculty members do regular observations and coaching sessions of teachers, and run PD and other collaboration meetings where teachers analyze student evidence. They also meet regularly with school administrators, bringing them into the PD cycle so they are part of the team supporting teachers.

Kathy Clemmer, the founding architect of CMAST, shared that while working with pre-service teachers, she noticed that “when they got hired and were integrated into the existing learning culture, all of the ‘innovative methodology’ they learned (in the pedagogical methods classes) disappeared.” Ms. Clemmer attended LMU in 1994 to earn her teaching credential, and subsequently became the first secondary math methods professor for what was then the Department of Education in 1997, while also teaching math at the nearby El Segundo High

School. As the Department of Education became the School of Education in 1998, Ms. Clemmer began teaching full time at LMU. She reflected on professional development (PD) experiences she had as a high school teacher, and noticed that most PD opportunities were geared for elementary teachers, and none were for math teachers. Additionally, PD opportunities occurred in a workshop format with no follow-up or continuing support. When the Dean of the newly minted School of Education asked her to write a grant to implement a secondary math and science professional development program for a local charter high school, the Math and Science Teaching (MAST) system was born. The success of this grant from the Stuart Foundation led to the founding of the Center for Math and Science Teaching (CMAST) within the School of Education at Loyola Marymount University, and the support given to in-service teachers by the center became known as CMAST. CMAST had been in existence since 2007 and, to date, had worked with over 85 schools in 25 districts, charter, and private school organizations.

Ms. Clemmer's vision was to provide professional development grounded in research, specifically to secondary math teachers, in order to grow their practice as professional educators by connecting various recommended teaching strategies. With the adoption of the Common Core State Standards in Mathematics in 2010 (www.corestandards.org), the Center refined its training around the Standards for Mathematical Practice in order to more adequately prepare teachers to deliver Common Core Instruction. The instructional methods that participating teachers were trained to use was called Mathematics Learning by Design (MLD), although in the field the name CMAST was still better known.

Mathematics Learning by Design (MLD)

The pedagogical techniques utilized by the CMAST staff to train teachers were called Mathematics Learning by Design (MLD). At its core, the MLD system was inspired by three perspectives: Lee Shulman and his work on Pedagogical Content Knowledge (PCK, 1987); Peter Senge and his work on collective intelligence and collaborative thinking systems (Senge, Lichtenstein, Kaeufer, Bradbury, & Carroll, 2007); and Bransford, Brown and Cocking's 1999 book, *How People Learn,* for exploring brain-based learning, primarily active learning and metacognition. The MLD system begins by utilizing the work of Melanie Green (2004), asking teachers to think about their content and content standards as a story, identifying what the climax of the story would be, and thinking about topics in their course as plot points that build toward, or fall from, that climax. Once the story has been shaped, teachers write learning targets based on the content standards. These learning targets are aligned to content standards, describe what students should know and be able to do, and are written in student-friendly language so that the students can use them to track their own learning. Teachers are then asked to rethink the traditional grading system by using a combination of standards- and mastery-based grading that involves assessment based on learning targets. All learning targets are graded on a rubric (usually using 4 points); achievement grades are based only on assessments. Teachers learn to "spiral" content, or periodically re-assess old learning targets and replace old grades with the most current demonstration of mastery.

All of this classroom procedure sets the stage for the actual pedagogy, which is based around five "instructional moves" (Clemmer & Laskasky, 2014): (a) Hook, (b) Investigation Before Explanation (IBE), (c) Interactive Notes, (d) Active Practice, and (e) Action Plan. The primary driver of student learning is inquiry, with the IBE ideally being a problem just beyond what the students can solve without learning more content, thus driving the need for content knowledge. Students are given a tool called ACE'M developed by Clemmer, Phillips, McCallum and Zachariah (2014, see Appendix A) based on Polya's (1962) problem solving framework to help students develop their skill as problem-solvers. The purpose behind the ACE'M tool is to support students through the different stages of the problem-solving process. ACE'M stands for Approach, Create, Execute, and Monitor; students approach a problem by determining what the problem is asking them to do, they create a plan to solve it, then execute that plan. Monitoring takes place throughout the process as students are encouraged to check frequently along the way that their approach and plan are aligned to what the problem is asking. Students also track their progress using learning target logs, which are a list of learning targets for each unit with a place for students to record their scores on assessments, and use that information to make decisions about what to practice, with support from the teacher. In short, the MLD system places students at the center of their own learning by using learning targets to track their learning progress, while teachers constantly provide opportunities for students to demonstrate their mastery of content, using the framework of a story to communicate and teach content.

The CMAST Teacher Leader model. Typically, local districts, charter management organizations, or diocese administrators contact CMAST at LMU to request training for their teachers. University faculty from the Center then conduct a meeting with the administration at the district, charter, or Archdiocesan level to present the program and discuss time and financial commitments. After securing permission from the administration, CMAST faculty hold "open" meetings for any interested schools, including site administrators and teachers. In order to engage with a school site, the site must identify at least two interested in-service math teachers who volunteer to work with a Teacher Leader (TL). The TL is a teacher identified at a school site by site administration and CMAST faculty from LMU, and coached by CMAST faculty. TLs also attend three semesters of coursework at LMU over two years, and are trained on adult learning theory, coaching adults, building effective professional development, and working with administration and community partners. The in-service teachers learn alongside the TL, and subsequently are coached by the TL.

Professional development. The TLs and district teachers all participate in a monthly cycle of Professional Development (PD), run by University CMAST faculty. These PDs include classroom observations of teachers, debrief meetings with teachers, and a department meeting of all teachers, run as a Professional Learning Community (PLC), where the focus is on analyzing student work around that month's PD topic. Each PD is focused on one of the MLD instructional moves. Move 1 is the Hook, where teachers use something to engage student interest. This can be a movie clip, picture, carefully posed question, or anything where students will be engaged and have to make connections to the content. Move 2 is the Investigation Before Explanation (IBE), in which students are presented with a problem that they do not yet have all the necessary skills or information to solve. Teachers support the students by using strategically written questions that help drive inquiry without solving the problem for the students, and either helping the students find what they need as they need it, or providing it for them if it is something beyond the students' ability to find. Move 3 is Interactive Notes, where the emphasis is on providing structure and opportunity for students to construct their own meaning and connect to their prior knowledge while creating notes. Move 4 is Active Practice, in which students are able to practice problem solving and computation while interacting with each other, checking their answers, and moving up in difficulty level as they are able. Move 5 is Goal-Setting and Action Plans, and teaches students how to set learning goals and write an action plan to achieve them with the support of the teacher. Assessment, both formal and informal, takes place throughout each instructional move and both helps the teacher plan instruction as well as helping students drive their learning.

In a typical monthly PD, for example, teachers might learn about how to engage students in an IBE. The PD will introduce teachers to what the IBE is, show how it works with students, then give the teachers strategies to help them be successful with using it, such as providing scaffolding questions for students and assessing when to stop the students and provide more information. Teachers are given time during the PD to choose an IBE problem and plan how to incorporate it into their practice, with the expectation that they will implement what they have planned in the next two weeks. About two weeks after the PD, the teachers bring samples of student work to a PLC, where the emphasis is on analyzing the students' work to see if the learning objective has been met. If it has, next steps are planned. If it has not, teachers will trouble-shoot and plan how to support the students in mastering the learning objective. Teachers also have the opportunity to ask questions and refine their understanding of the instructional move they have just learned.

The goal of CMAST, in working with TLs to master these instructional moves, is to create a sustained pedagogical practice grounded in MLD. As such, TLs, who are completing university coursework while participating in the program, gradually assume more coaching responsibility with their colleagues. They learn to facilitate the PLC and develop their own PDs.

Because of the length of time it takes to train a TL, most projects are supported for a minimum of two years, and CMAST support will often continue on an as-needed basis once the TL has been certified and is operating independently.

This system stands in sharp contrast to other professional development models in several ways. First, CMAST begins with identifying a leader from within the school site, who is already a part of the school site's culture, and who will become the change agent moving forward. In that way, the school is not dependent on an outside source of support and inspiration for change. Second, the CMAST professional development takes place over two years with regular monthly follow-up including in-class observations and instructional rounds, where a group of teachers is led by either CMAST faculty or a TL into classrooms to observe teachers demonstrating the instructional moves. Finally, the instructional moves that are taught to teachers are connected systematically so as to compel a teacher to rethink all parts of what happens in their classroom, rather than being a new component that the teacher would plug into existing pedagogy or classroom practice.

Personal Connection

I have been a teacher since 1997, when I began my teaching career in a first-grade classroom in a large urban K–8 district in Central California. I noticed immediately that there was a great deal of emphasis placed on literacy, and I was required to attend frequent literacy trainings, but never offered math training. After three years teaching elementary grades, I moved to Southern California, where the only option was a middle school position teaching sixth grade. In my second year, the entire sixth grade math team retired, so I offered to teach math and I have been a math specialist ever since. I began with an elementary multiple-subject credential as my undergraduate degree was in music, and eventually earned a foundation-level math credential. After a few years in sixth grade, I went on to teach seventh-grade math, and Algebra 1, and became a math resource teacher for the district in which I currently teach. In addition to the EdD program, I am also finishing a Master of Arts in Teaching Mathematics. I have always been aware of the problems that arise when math content knowledge is limited, so this study is of deep personal significance to me.

My involvement with CMAST began in the Fall of 2010 as the Center was transitioning to Common Core implementation. I trained as the Teacher Leader at my school site, a public middle school with over 800 students of various racial and ethnic backgrounds in the greater Los Angeles area, and eventually began to coach my math department. In 2012, I was invited to work for CMAST, as university faculty, which I have done through the present (2017). Currently, I support a network of 25 certificated math and science TLs for grades three through 12 in the same district in which I still teach an Algebra 1 class at a middle school. I also work with a neighboring district where I am training 5 TLs in two middle schools. I have been working with this neighboring district for two years, as they are gradually rolling out the program to all of their math teachers.

When I first began participating in CMAST PDs in 2010, the session usually involved about 20 math teachers from three middle schools. Teachers were asked to complete a typical math problem that students would be asked to complete in class. Sometimes teachers were given two problems differentiated for different grade levels. There was often time reserved to discuss what student misconceptions about the problem might be. Eventually, however, the PDs stopped including this content-based component and became primarily about pedagogy. Content knowledge was always a baseline, especially when discussing how to choose a good IBE task, for example, but the explicit content piece was no longer part of CMAST PDs. I started to notice that teachers with less content knowledge struggled, and were less willing to plan lessons or inquiry experiences outside of what was provided by their textbook. During a CMAST PD I was leading, a teacher told me that, “the textbook knows better than I do” when describing a “good” math problem. Other teachers struggled, especially after Common Core was adopted, with addressing what conceptual understanding might look like for students, as they claimed that they were “never taught to think about math this way.” These experiences led me to believe that content understanding must be connected to pedagogy in order to have the biggest impact on teaching practice. Daniel Willingham (2008) pointed out that critical thinking is not a skill that can be taught devoid of context, and I believe that applies to planning pedagogy as well critical thinking and problem-solving.

The Current Study PD

Based on my experiences with CMAST, I tapped into my own current practice as CMAST faculty and designed a content-based PD to be dropped into a current CMAST PD offered in my neighboring district. I chose ratios and proportions as the topic for the content portion of both PD sessions because ratios and proportions are a domain specific to grades six and seven in the Common Core mathematics standards, and they are essential to higher-level math understanding, yet they are difficult to teach and many middle school students struggle with them (Lobato & Ellis, 2010). The group of teachers to whom I administered PD were all middle school teachers, most of whom did not have full math credentials. The majority of them had some exposure to the instructional moves of MLD, so for the 2016–2017 school year, we were going to be refining some of their classroom practices and going deeper into one or two of the moves as requested by the TLs.

Session One

The topic of the August 30, 2016, PD was scheduled to be on designing good assessment questions and giving students feedback. I opened the PD by having the teachers do a math task using proportional reasoning so I could assess their understanding before we discussed any content. I made the first half of the PD a content session on ratios and proportions, starting with having the teachers articulate their beliefs about whether ratios and fractions are the same, giving them some data about the performance of middle school students on a proportional reasoning task, and emphasizing the difference between reasoning with one variable versus reasoning with two, which is what is called for in proportional reasoning. Teachers were then asked to answer a simple proportion question and draw a model to show their reasoning. After analyzing a student response to the question, and examining her reasoning, we discussed the different ways in which ratios represent data, compared and contrasted ratios and fractions, and examined the meaning of some ratios in context. We then moved into the pedagogy portion of the PD, and I explicitly connected it to the ratio and proportion section when discussing assessment questions. The objective was to design or choose assessment questions that revealed student thinking, and to use that thinking to gives students feedback as feedback has been positively associated with student growth in mathematics (Boaler, 2016; Deevers, 2006; Elwar & Corno, 1985). When discussing what good assessment questions looked like, I referred back to the ratio questions in the content portion of the PD, and we examined them for their potential to reveal student thinking. The PD finished with planning time for teachers to find or design their own assessment questions, and as school was beginning for them the following day, they focused on whatever the content was for their first unit. (See Table 3).

Table 3

Sample Plan for Study PD #1

Time	Activity	Desired Outcomes
5 min.	Welcome and Sign-in	Lower affective filter
10 min.	Content question and discussion	Clarify existing beliefs about content
15 min.	Presentation/discussion of student samples	Explore student misconceptions
5 min.	Presenter teaching	Clarify participant understanding
10 min.	Participant modeling	Have participants experience as student
10 min.	Participant/student samples	Examine how students approached models and identify misconceptions
5 min.	Question for discussion	Clarify existing beliefs about assessment
15 min.	Samples for analysis	Participants compare/contrast samples
5 min.	Revisit earlier samples	Analyze student thinking
10 min.	Question for discussion	Build consensus on effective feedback
20 min.	Team planning time	Content teams plan how to implement ideas from PD
10 min.	Collective agreements and survey	Team agrees on evidence to collect and gives presenter feedback

Note. Times given are approximate as presenter must assess in real time and make adjustments where necessary.

Session Two

The topic of the second PD, which occurred seven weeks later on October 18, 2016, was to explore the IBE more deeply. While most of the participants had been exposed to the idea of it, asking them to give students a problem they could not fully solve in order to promote inquiry was something most of them were not yet comfortable with. This PD also opened with an exploration of ratios and proportions, with the emphasis this time being more specifically on proportional reasoning. Teachers were given scenarios that all seemed proportional, though some were not. They identified which were proportional and brainstormed possible student misconceptions and errors that would occur. I then gave them suggestions for how to promote proportional reasoning without immediately moving to cross-multiplication, which is a typical shortcut that can bypass conceptual understanding. Then we looked at contexts that would require different types of proportional reasoning and discussed strategies for promoting it in students. We then moved into an exploration of the IBE in which I showed teachers a movie clip about Voyager 1 and asked them to calculate how far it currently was from Earth. I modeled asking them supporting questions, and we discussed how this type of inquiry illustrated the connection between the curriculum, students' prior knowledge, and the world which is crucial for learning (Bransford et al., 1999). I explicitly connected it to the content portion of the PD by asking teachers if there was any way to incorporate proportional reasoning into their plan to solve the problem. The PD then closed with planning time for teachers to design their own IBE and teachers doing another proportional reasoning problem to assess their knowledge after two content PDs (See Table 4).

Table 4

Sample Plan for Study PD #2

Time	Activity	Desired Outcomes
5 min.	Welcome and Sign-in	Lower affective filter
10 min.	Content question and discussion	Clarify existing beliefs about content
15 min.	Presentation/discussion of student samples	Explore student misconceptions
5 min.	Presenter teaching	Clarify participant understanding
10 min.	Situational Analysis	Have participants identify when a given situation is proportional
10 min.	Task Analysis	Examine tasks and predict student misconceptions
20 min.	Investigation activity	Participants experience constructivism
15 min.	Examine guiding questions	Participants learn to support students while honoring prior knowledge
5 min.	Revisit content topic	Connect content and pedagogy
20 min.	Team planning time	Content teams plan how to implement ideas from PD
10 min.	Collective agreements/exit interviews	Teams agree on evidence to collect and give presenter feedback

Note. Times given are approximate as presenter must assess in real time and make adjustments where necessary.

The following chapters will discuss results of the study, analysis, and discussion of those results, and recommendations for next steps.

CHAPTER 5

FINDINGS

Study Background

The Center for Math and Science Teaching (CMAST) at Loyola Marymount University (LMU) was a "program committed to developing, inspiring, and retaining excellent STEM teachers" (http://soe.lmu.edu/centers/cmast/). CMAST trained teachers and Teacher Leaders in 25 schools representing 11 districts, charters, and diocesan organizations throughout Southern California through a monthly cycle of Professional Developments (PD), where teachers were trained in inquiry-based pedagogy, and Professional Learning Communities (PLC) in which teachers met to analyze student evidence collected from the pedagogy learned in the monthly PD. As the focus of CMAST was changing teacher pedagogy, the current study sought to examine the impact of adding a content piece, as research has shown that teacher content knowledge has an impact on their instruction (Heilig & Jez, 2010; Raymond et al., 2001; Telese, 2012). The interplay of teacher self-efficacy, content knowledge, and adopting a more constructivist approach to teaching math need to be understood for CMAST to increase its impact on teachers.

The purpose of this study was to examine the ongoing CMAST PD on mathematics pedagogy and highlight the impact of embedding content-based PD within it. To measure the impact of the two content-based PD sessions, created for this study and embedded within a larger PD program for math teachers, participants completed a pretask before the first PD and a post-task after the second PD to assess content knowledge. To measure teacher self-efficacy toward teaching math, participants completed the Math Teaching Efficacy Belief Instrument (MTEBI) survey before the first PD and again after the second PD, and to measure intended classroom teaching practices, participants answered a series of exit interview questions after the second PD.

Study Participants

Twenty secondary math teachers participated in this study. All of the participants were employed as teachers at two middle schools in a small urban school district in Southern California, and during this current school year (2016-–2017), teaching sixth-, seventh-, or eighth-grade students. There were 18 female participants, and two males, and their number of years of teaching experience ranged from one to 24, with a mean of 8.3 years, and modes of six and seven. Nine of the participants held an elementary multiple-subject credential, one had an elementary credential and a middle grades (5-9) math authorization from the State of Florida, four had an elementary credential and a Foundation Level math credential (which authorized them to teach math content through geometry, typically a course for 10th-graders), two had an elementary credential with a supplemental authorization in math (indicating that they had completed an additional 20 units of math coursework beyond what was required for their nonmath undergraduate degree), and four held a full single-subject math credential. Math courses taught by the participants in the year prior to the study ranged from fifth grade, to AP Calculus AB. I had been providing math pedagogy PD to eight of these teachers for the previous two years, seven of these teachers joined our PD session in the previous year, and four of the teachers met me for the first time during this study's first PD session in August 2016.

Professional Development Design

The standard format for the typical CMAST PD for math teachers was a two-hour session using a PowerPoint presentation with a mixture of teaching slides, discussion facilitation slides, and activity slides that prompted teachers to clarify their existing beliefs around a topic, give new information about that topic to inspire cognitive dissonance, and/or actively produce something or reflect. The objective of the PD was pedagogical in nature with teachers being taught a new pedagogical approach to content, and asked to try the new approach. For the purposes of the current study, the newly created PD sessions focused on math content rather than on pedagogy. For these two content-based PD sessions, I spent the first hour of each PD focusing on content and devoted the second hour to pedagogy, connecting the content and pedagogy explicitly in the second half of the PD. I chose to focus the math content on ratios and proportional reasoning. I chose this area as it is a developmentally crucial concept in middle school, which leads to understanding of functions and algebraic concepts as students transition into high school math. Additionally, ratios and proportions are a content domain, which exists solely and specifically in sixth and seventh grade in the Common Core math standards. Given that the teachers are all middle school teachers, the content was an appropriate focus area for these grade levels, and proportional reasoning is typically underdeveloped in middle school students (Lobato & Ellis, 2010).

Content Knowledge

At the beginning of the first PD on August 30, 2016, participants were asked to complete a math problem or task developed by the Silicon Valley Mathematics Initiative (SVMI), which asked them to print a photograph and two smaller copies on a rectangular sheet of paper. The task then showed two different possible arrangements of the photos with partial measurements and asked participants to give the missing measurements. To properly solve this task, participants could use their understanding of ratios and proportions. As such, this pretask provided a baseline of their content knowledge prior to the content-based PD on ratios and proportions. The post-task was given at the completion of the second PD on October 18, 2016. This task was another ratio and proportion task provided by the SVMI, and asked participants to find unit rates for two different people mowing a lawn. Participants were then asked to calculate how long it would take the lawn to be mowed if the two of them worked together. Again, ratios and proportions could be used to solve this task, providing a sense of change in content knowledge from before to after the PD.

Tasks were labeled by participant ID number and scored by two nonparticipant math teachers from a local school district; in order to minimize bias, the participants were unknown to the graders. The tasks were scored using a rubric provided for the task by the SVMI. To establish inter-rater reliability, graders first scored the same task independently using the rubric and then discussed their use of the rubric to calibrate their scoring procedures. Each task was then scored twice, once by each grader, with any scoring discrepancies examined and rescored until agreement was reached. Across the board, graders maintained a sense of consistency, with only two tasks needing to be discussed. Figures 1-10 presented below show the tasks completed by the participants. Work samples completed by the participants were also provided and showcased how teachers varied their problem-solving approach.

Pretask. Before the PD occurred, participants were asked to do a task that required them to scale up copies of photographs. Figure 1 below shows the pre-task.

Photographs

This problem gives you the chance to:

- use proportion in a real life geometric context

A photographer wants to print a photograph and two smaller copies on the same rectangular sheet of paper. The photograph is 4 inches wide and 6 inches high.

Here are two ways he could do it. (Note: the diagrams are not drawn to actual size.)

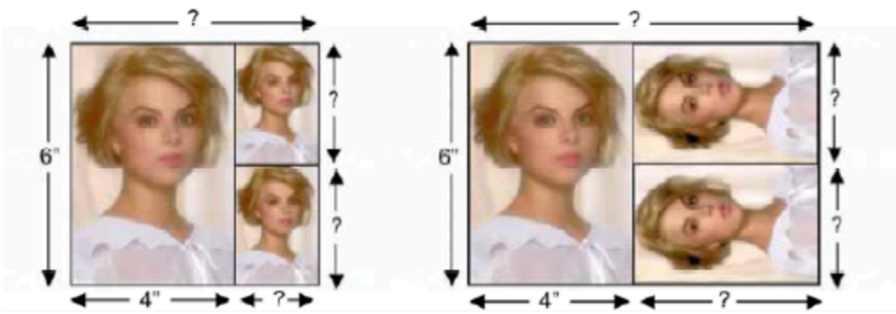


Diagram 1 **Diagram 2**

1. Find the measurements of the small photographs for each arrangement.
   Show your calculations and explain how you figured it out.

2. Find the size of the sheet of paper for each arrangement.

   Diagram 1

   The measurements of the sheet of paper are ___________ wide and ___________ high.

   Diagram 2

   The measurements of the sheet of paper are ___________ wide and ___________ high.

Figure 1. The pretask.

The pretask seen above was designed to be solved using proportional reasoning, with the anticipated result that participants would set up a proportion to find the missing measurements. Figure 2, below, shows the rubric used to assess the task.

Photographs		Rubric
The core elements of performance required by this task are: • use proportion in a real life geometric context Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Diagram 1: The height of the smaller copy = 1/2 of 6 inches = **3** inches	1	
Uses proportional reasoning correctly: Height/width = 6/4 = 3/width **or** Size of photo/Size of copy = 6/3 = 4/width Width = **2** inches Accept verbal reference to scaling if answer correct.	1 1	
Diagram 2: The width of the smaller copy = 1/2 of 6 inches = **3** inches	1	
Uses proportional reasoning correctly: Height/width = 6/4 = height/3 Height = **4 1/2** inches Accept verbal reference to scaling if answer correct.	1 1	6
2. Gives correct answers: Diagram 1: **6** inches wide, **6** inches high Diagram 2: **8.5** inches wide, **6** inches high	 1 1	2
Total Points		**8**

Figure 2. The pretask grading rubric.

As seen in Figure 2, above, the task is designed to be answered using proportional reasoning, and the rubric gives an example of the proportion. Figure 3 shows an example of a sixth-grade teacher with a Multiple Subject Credential solving proportionally, although she did not show the proportions as fractions.

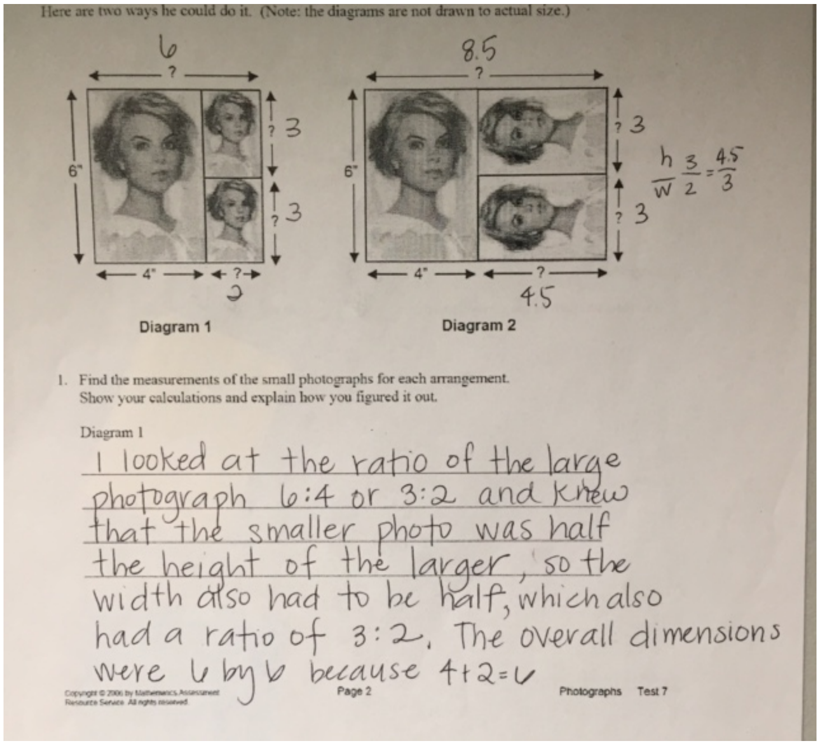


Figure 3. A sixth-grade teacher with a Multiple Subject credential reasons proportionally.

In Figure 4, below, an eighth-grade teacher who had a full Single Subject credential attempted to set up an equation and reasons additively as seen in the work at the very bottom of the page. Eventually, she switched to a proportion as can be seen on the right and left side of the photographs; this was especially interesting as she described that she needed to set up proportions to solve, but appeared to attempt the equation first.

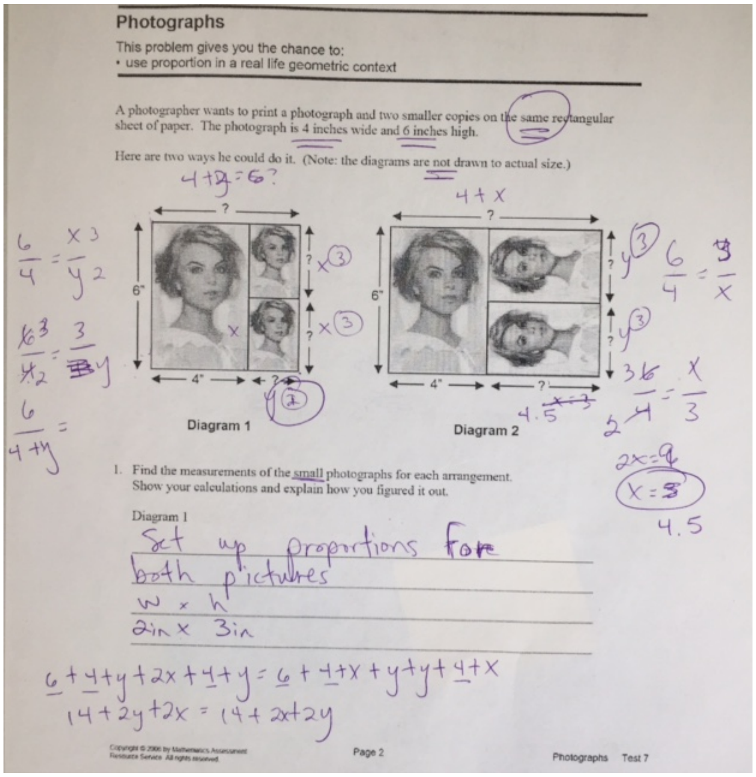


Figure 4. An eighth-grade teacher with a Single Subject credential attempts to reason algebraically, then switches to proportional reasoning.

In Figure 5, below, an sixth-grade teacher with a Multiple Subject Credential and a middle school authorization solves the second problem in the task, but appears to be reasoning additively, leading her to an incorrect answer of 9 rather than 4.5.

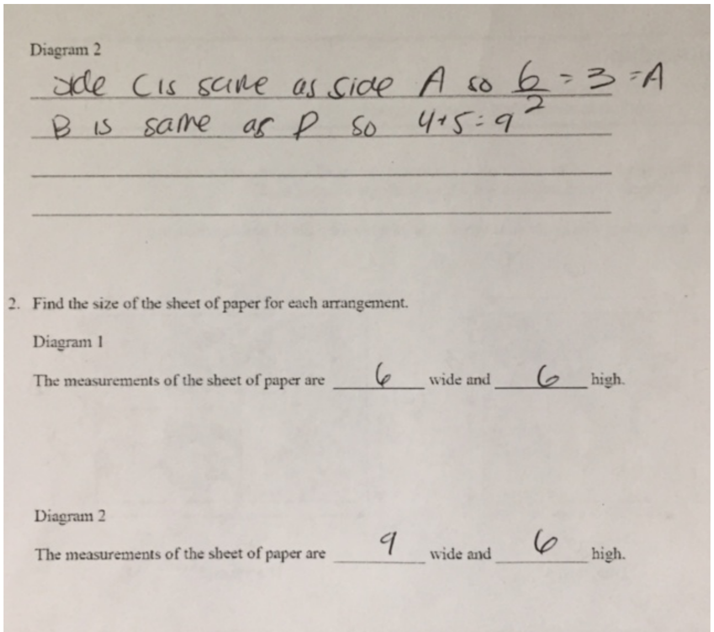


Diagram 2

side C is same as side A so $\frac{6}{2} = 3 = A$

B is same as D so $4+5=9$

2. Find the size of the sheet of paper for each arrangement.

Diagram 1

The measurements of the sheet of paper are 6 wide and 6 high.

Diagram 2

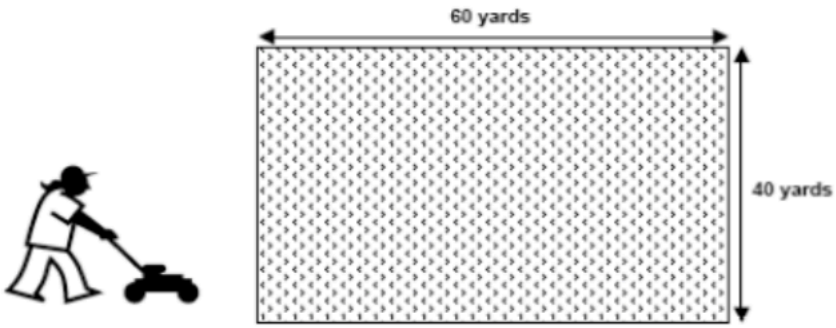
The measurements of the sheet of paper are 9 wide and 6 high.

Figure 5. A sixth-grade teacher with a Multiple Subject credential + other authorization reasons additively rather than proportionally.

Post-Task. After the second content PD, participants were asked to solve a problem requiring them to figure out how long it would take for two people to mow a lawn together after they had been guided through the calculations necessary to determine the amount of lawn per minute each person could mow. Many of the study participants expressed the opinion that this task was more difficult than the first task, although scores on this task were higher in general. Figure 6, below, shows the post-task.

Lawn Mowing

This problem gives you the chance to:
- solve a practical problem involving ratios
- use proportional reasoning



Dan and Alan take turns cutting the grass.
Their lawn is 60 yards long and 40 yards wide.

1. What is the area of the yard? ______________ square yards

Dan takes an hour to cut the lawn using an old mower.

2. How many square yards does Dan cut in a minute? ______________
   Show your work.

Alan only takes 40 minutes using a new mower.

3. How many square yards does Alan cut in a minute? ______________
   Show your calculation.

4. One day they both cut the grass together.
   How long do they take? ______________

Figure 6. The post-task, requiring participants to determine how long it would take two people to mow a lawn together after determining each person's rate per minute.

The post-task shown above was designed to be solved proportionally, although participants did not have to set up a formal proportion to solve. Figure 7, below, is the rubric.

Lawn Mowing Grade 7		Rubric
The core elements of performance required by this task are: • solve a practical problem involving ratios • use proportional reasoning Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **2,400** square yards	1	1
2. Gives correct answer: **40 square yards per minute** Shows work such as: (60 x 40) ÷ 60	1 1ft	2
3. Gives correct answer: **60 square yards per minute** Shows work such as: (60 x 40) ÷ 40	1 1ft	2
4. Gives correct answer: **24 minutes** Shows correct work such as: In one minute together they mow 40 + 60 = 100 square yards (60 x 40) ÷ 100	1 2ft	3
Total Points		**8**

Figure 7. The post-task rubric.

As shown above, the rubric indicates that the expectation is to use proportional reasoning, although there was no explicit expectation that participants would set up a proportion. Figure 8, below, shows the work of an eighth-grade teacher with a full Single Subject math credential who arrived at a solution algebraically by creating an equation whereby each person's rate was expressed as a separate ratio, rather than adding the two rates together and dividing at the end.

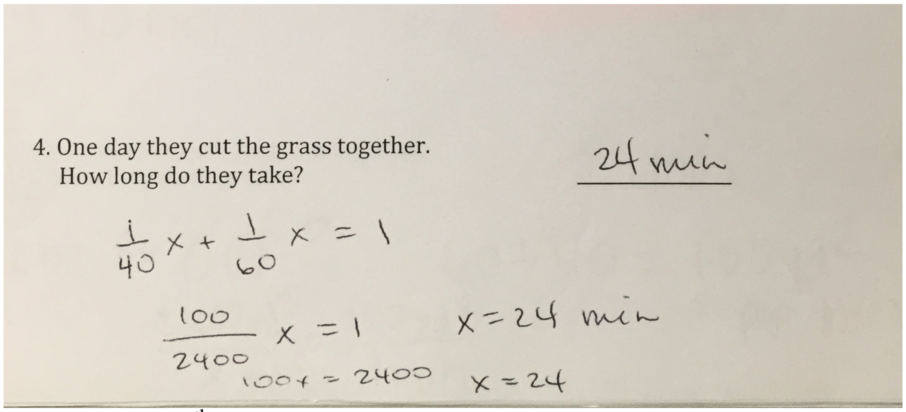


Figure 8. An 8th grade teacher with a Single Subject credential solves algebraically.

As can be seen in Figure 8, this teacher rendered each person's rate as a separate fraction, then found a common denominator and added, then multiplied both sides of the equation by 2,400 to clear the denominator. The teacher whose work is shown in Figure 9, below, did essentially the same thing, except he or she simply multiplied 60 by 40 to find the amount each person would mow, then divided by the combined rate of both people.

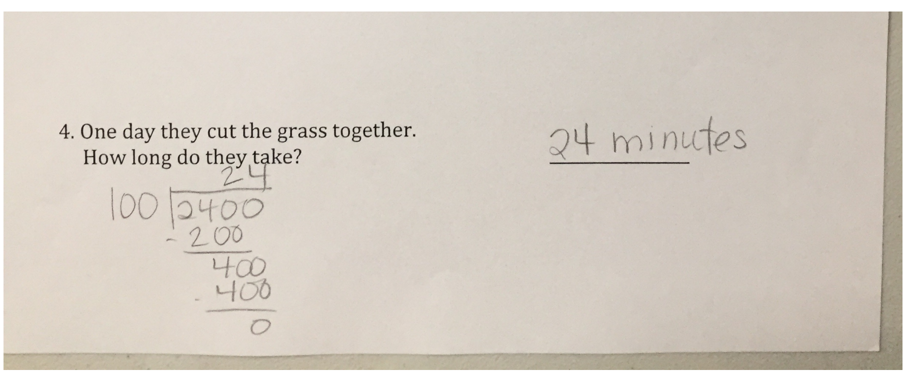


Figure 9. A sixth-grade teacher with a Multiple Subject credential reasons proportionally. Notably this teacher solved essentially the same way as the teacher in Figure 8 but without the algebra.

Figure 10 also shows a sixth-grade teacher with a Multiple Subject Credential, but this teacher used additive reasoning rather than proportional reasoning. While the structure of her work looks similar to that in Figure 9, she chose to split the total area of the lawn in half, then divide by each person's unit rate and add at the end, missing that since their individual unit rates differ, this approach won't work.

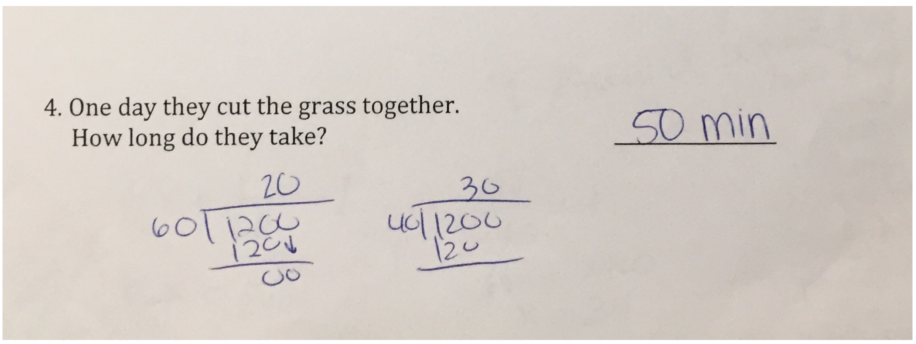


Figure 10. A sixth-grade teacher with a Multiple Subject credential reasons additively rather than proportionally.

Both tasks were worth a total of 8 points, and 18 valid scores were collected on both. The mean score on the pretask was approximately 6.06 with a standard deviation of 2.04, and the mean score on the post-task approximately 7.33 with a standard deviation of 1.19. A dependent samples t-test of the pre- and post-tasks indicated a small but significant increase in content knowledge of ratio and proportions [t(17) = -2.44, $p < .05$, Cohen's d=0.91], suggesting that the PD session had some positive impact on the participants' content knowledge. Looking at mean scores by credential type also showed growth in each category with the exception of Multiple Subject + other Credential, which was a category of one person whose score did not change. (See Table 5, below)

Table 5

Mean task scores by credential type

Credential Type	Number	Pre-Task	Post-Task
Full	4	7.3	7.7
MS +Found.	3	6	7.3
MS+ Supp.	2	3.5	8
MS + Other	1	5	5
MS Only	9*	6.1	7.3

Note. MS = Multiple Subject; Found = Foundation level math; Supp = Supplemental.
*Only 8 scores were considered valid in this category

The credential types in the table above include a Full Single Subject math credential (teacher has an undergraduate degree in math), Multiple Subject only (teachers who have been credentialed to teach elementary math, but in California are also allowed to teach sixth grade in a middle school), Multiple Subject + Foundation Level (Elementary credential with a subject matter exam that allows teaching up through 10th-grade Geometry), Multiple Subject + Supplemental (Elementary credential with 20 units of additional college math courses) and Multiple Subject + other (Elementary credential with a Middle School authorization from Florida).

Teacher Self-Efficacy

Of the twenty participants, nineteen took both a pre-and post-survey to measure their self-efficacy toward teaching math. The presurvey was given at the beginning of the first PD on August 30, 2016, and the post survey, which was an identical copy of the presurvey, was emailed to participants the day after the second PD on October 18, 2016. By October 30, 2016, all participants had completed the post-survey and the survey was closed. The survey was the

Mathematical Teaching Efficacy Beliefs Instrument (MTEBI), developed and validated by Enochs et al. (2000). The MTEBI measures two variables; Outcome Expectancy, which according to Gibson and Dembo (as cited in Enochs et al., 2000, p. 195) is defined as teachers' belief that student learning can be influenced by effective teaching, and Personal Math Teaching Efficacy Belief, which Enochs et al. have defined as a belief in one's ability to teach mathematics effectively. Cronbach alphas were examined among the current sample and both Outcome Expectancy (alpha = .77) and Personal Math Teaching Efficacy Beliefs (alpha = .74) had adequate internal reliability. Outcome Expectancy correlated positively from before to after the PD ($r = .67$, $p < .01$), suggesting temporal stability. Personal Math Teaching Efficacy Beliefs correlated positively from before to after the PD ($r = .69$, $p = .001$), suggesting temporal stability as well.

A mean composite variable was created for Outcome Expectancy and for Personal Math Teaching Efficacy Beliefs and dependent-samples t-tests were run on each composite to examine change from before to after the PD. There was not a statistically significant difference between the pre-and postsurveys for Outcome Expectancy ($t(18) = -0.19$, $p = 0.85$) or for Personal Math Teaching Efficacy Belief ($t(18) = -0.35$, $p = 0.73$), which would indicate little-to-no change in teachers' self-efficacy toward teaching math during the seven-week period of the study. Table 3 gives the surveys items that measured Outcome Expectancy, and Table 4 gives the items that measured Personal Math Teaching Efficacy Beliefs.

Intended Teaching Practice

To measure teachers' intended classroom teaching practices after participating in the content-based PD sessions, at the conclusion of the second PD on October 18, 2016, after finishing the post-task, participants were asked to step into one of four side rooms and use an iPad to record their responses to interview questions, assessing the extent to which the PD would impact their teaching practice. The interview questions asked were:

1. Will today's learning have any impact on your teaching practice? If so, how? If not, why not?

2. What is a question you still have after today's learning?

3. What is something you would like more support with in your math teaching?

4. Do you have any other questions or thoughts to share with the researcher?

Of the 20 participants interviewed, 19 indicated that the content of the PDs would impact their teaching practice. These participants responded to the question by simply indicating that the PD would impact their practice, and a few chose to elaborate by describing how their teaching practice would change. Although the questions asked about the last two content-based PDs, 18 of the respondents spoke specifically about the PD they had just completed in October and referred to the pedagogical content only. Interestingly, only one respondent spoke specifically about the ratio and proportion content and indicated that although it was not a content standard she taught, she felt it would somewhat impact her practice. She shared, "I think looking at proportional relationships in different ways would be helpful for my students to have a better understanding of proportional relationships. Right now I think I'm looking at it in a pretty narrow perspective."

To examine responses through the lens of constructivist theory, I attempted to code responses for an indication of changing practices to be more inquiry-based. To this end, each affirmative response to the question of whether the PD would have an impact on their practice was rated on a scale of 1–3, with 1 indicating a simple affirmative "yes" response, and 3 indicating an affirmative response with specific examples of what they would put into practice. Given that nearly all of the respondents referred to the most recent PD and its exploration of how to use inquiry to drive a math lesson, the behavior in question was implementing inquiry. Two of the participants gave a response rated as a one, nine gave responses rated as a two with some ideas about what they might do—but nothing specific—and seven gave responses rated as a three, with specific examples of how they intended to implement a more inquiry-based pedagogy. For example, one participant noted that she would “like to try to incorporate more tasks that allow those students to explore instead of me just kind of feeding them or doing too much direct instruction,” while another stated she will spend more time “just having the kids explore rather than doing too much direct instructions,” clearly indicating they wanted to attempt a more inquiry-based approach. One participant mentioned that he or she now had a better understanding of inquiry and how “they’re [the students] kind of teaching themselves and I’m guiding them,” and another mentioned, “I really like the IBE layout and that it’s having the children think and ask questions before you dive in and give them all the answers.” Many of the participants mentioned that they would like further support with either finding or implementing good tasks to promote inquiry, which indicates both their willingness to try a more inquiry-based approach, as well as a need for further support.

Conclusion

Overall, the participating teachers’ content knowledge about ratios and proportional reasoning increased slightly over the seven-week period between the first and second PDs. Exit surveys indicated that the most recent PD session would have an impact on their teaching practice, although the impact would mostly be related to their pedagogy. When it came to teacher self-efficacy, however, the results of the pre- and post-surveys indicated little change in teachers' self-efficacy toward teaching mathematics. Possible reasons for some of these results, as well as recommended next steps, will be the focus of Chapter 6.

CHAPTER 6

DISCUSSION

Professional Development (PD) for teachers is important enough that the federal government requires that 10% of Title I funds be used for it (Mizell, 2010). According to Darling-Hammond et al. (2009), “well-designed professional learning helps teachers master content, hone teaching skills, evaluate their own and their students’ performance, and address changes needed in teaching and learning” (p.7). In order to address those needed changes, teachers have to be able to create an effective learning environment. Bandura (1997) stated that “the task of creating learning environments conducive to the development of cognitive competencies rests heavily on the talents and self-efficacy of teachers” (p. 240). Teaching mathematics effectively requires a delicate balance of content knowledge about math, Pedagogical Content Knowledge (PCK) about the way math and pedagogy interact, and Mathematical Knowledge for Teaching (MKT), which involves how students interact with the content. At the core of these is teacher self-efficacy toward teaching mathematics or “beliefs in one’s capabilities to organize and execute the courses of action required to produce given attainments” (Bandura, 1977, p. 3). As such, this study sought to examine the extent to which a math content-based Professional Development (PD) impacted math content knowledge, teacher self-efficacy, and teacher practices.

The study focused on middle school math teachers because many teachers at this grade level do not hold a full math credential (Harris et al., 2011), but are teaching what is considered secondary math. Two PD sessions were conducted approximately seven weeks apart. The PD sessions were administered as part of professional support given by the Center for Math and

Science teaching (CMAST) at Loyola Marymount University (LMU) for a small urban school district in Southern California; but, for the purposes of the study, the PD design was modified from the usual pedagogy-only focus to include targeted math content about ratios and proportional reasoning. Teachers completed pre- and post-math tasks on ratios and proportions to assess changes in content knowledge, pre- and post- surveys to assess changes in self-efficacy toward teaching math, and exit interviews after the final PD to assess the impact the PDs might have on their future classroom practice. Analyses of the data revealed that there was a small significant increase in teacher content knowledge but no significant change in self-efficacy. The PD was described by teachers as informative and likely to impact teachers' future pedagogy.

This chapter will discuss these findings and is organized into eight sections: teacher content knowledge, teacher self-efficacy, professional development impact, limitations, future research, implications, recommendations, and conclusion.

Discussion of Findings

Teacher Content Knowledge

Findings suggest that a content-based PD was effective in increasing math content knowledge specific to ratios and proportions. Research indicates that effective mathematics teachers need a combination of content knowledge, Mathematical Knowledge for Teaching (MKT) (Hill & Ball, 2009), and Pedagogical Content Knowledge (PCK) (Shulman, 1987). Embedded in both MKT and PCK is an assumption about teachers having the content knowledge necessary to teach their grade level standards. However, research indicates that in the United States more than half of fifth- through eighth-grade teachers are not certified to specifically teach math and were not math majors (Harris et al., 2011). This finding mirrors the dynamic for the majority of the teachers who participated in the current study; the majority held elementary credentials, but were teaching secondary math. Those teachers will need content support to teach effectively, as research indicates that content knowledge in math is connected to student achievement (Heilig & Jez, 2010; Raymond et al., 2001; Telese, 2012;). Since so many middle school math teachers do not currently hold full math credentials, providing them with PD that builds their content knowledge could be a way to close this gap.

Further complicating the issue of a teachers lacking content knowledge is that the Common Core State Standards for Mathematics were written based on research related to how students' understanding develops over time (corestandards.org). This means that teachers must not only deeply understand the content for their grade level or course, but must also situate that content in a larger vertical progression. For example, a sixth-grade teacher may not understand the purpose behind the standard that asks for students to: "Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates" (corestandards.org). The purpose behind this standard is twofold: developing student understanding of integer operations (adding, subtracting, multiplying, and dividing with positive and negative numbers) in seventh grade, and extending their understanding of the coordinate plane past the first quadrant which they have developed understanding of in fifth grade ("Progressions Documents for the Common Core Math Standards," n.d.). If teachers cannot see how this standard fits into the larger context of middle school math, they may decide that dealing with a simple number line is sufficient and ignore the part of the standard that deals with the coordinate plane, thus potentially setting students up for serious trouble in seventh grade as they examine linear associations by graphing them on the coordinate plane. This difficulty could potentially continue into eighth grade as they graph linear functions and construct geometric figures on the coordinate plane. As such, math content knowledge among teachers is critical given that standards are written for each grade level building on previous knowledge and because omissions in knowledge in earlier grade levels could contribute to difficulties with math in later grade levels.

Students exiting elementary school and entering secondary are coming from solidifying their understanding of fractions, and are often immediately shown ratios, notated as fractions, although their meaning is different. While ratios behave in some ways as fractions, in other ways they do not, and this difference is lost on the students, especially if the teacher cannot help guide them through it. As such, ratios and proportions was chosen as the math content for the study's PD given that one of the big changes from elementary to secondary math is the idea of covariance: a change in two quantities simultaneously, a concept illustrated by ratios, but not by fractions. Students' first exposure to covariance typically comes when learning about ratios and not being able to distinguish between ratios and fractions clouds the concept, potentially leading to problems with slope and rate of change as students enter Algebra (Lobato & Ellis, 2010, "Progressions Documents for the Common Core Math Standards," n.d.).

The math tasks given to the teachers to determine changes in their math content knowledge from before to after the PD were both performance tasks designed for seventh-grade students. Findings suggest a moderately significant improvement in math content knowledge from before to after the PD. Additionally, both tasks asked respondents to show how they had arrived at their answers, which provided a glimpse into the teachers' thought processes as problem solvers. The difference in how the teachers approached the problems showed a strong connection with their background knowledge; the single-subject teachers went straight to creating an equation, and the elementary credentialed teachers either set up proportions or attempted to use repeated multiplication or addition. There are advantages and disadvantages to each approach. Teachers who utilized an equation suggest more sophisticated abstract reasoning, but might not necessarily be able to help students make a connection to proportional reasoning. Teachers who used less sophisticated solution methods may suggest an understanding of student reasoning, as it more closely mirrors their own, but these teachers may struggle to guide students toward more abstract representations of problems. Overall, the analysis of pre- and post-tasks showed a small, but significant increase over the course of seven weeks, and while many of the teachers expressed the opinion that the post-task was more difficult, their scores improved and the reasoning shown in their work was more sophisticated.

Teacher Self-Efficacy

While there was no significant change in self-efficacy among teachers who participated in the math content-based PD, research indicates that the concept is still a critical component of effective teaching. Hattie (2008) stated that the teacher makes up 30% of the variance determining what influences learning the most. Bandura's (1977) definition of self-efficacy applied to teachers would indicate that teachers believe they have the ability to positively impact student learning with their actions. Research shows that teachers' self-efficacy about teaching can contribute to, or detract from, the success of their students (Althauser, 2015; Enochs et al., 2000; Henson, 2001; Swackhammer et al., 2009), which may be due to teachers with higher self-efficacy about their teaching having what Stanford psychologist Carol Dweck (2006) has referred to as a "growth mindset." A person with a growth mindset believes that abilities—including intelligence—can be increased through effort. A person with the opposite mindset—what Dweck referred to as "fixed"—believes that what you are born with is what you have, and there is no sense in trying to increase your intelligence as it is fixed. Stanford professor Jo Boaler (2016) took it one step further by connecting it explicitly to how students can use mistakes in math to create new synapses, thus enabling new learning. Teachers with a fixed mindset would believe that their students have fixed abilities in math and that mistakes are not useful. Their self-efficacy toward teaching is likely to be low as they would not believe that their efforts have much impact on student success. Teachers with a growth mindset would be more likely to believe that students can improve with effort, view mistakes as learning tools, and have higher self-efficacy toward teaching as they would be more likely to believe that their efforts in teaching would have impact on their students.

There are many possible reasons that self-efficacy did not change much over the course of the study. Bandura (1997) believes that self-efficacy is stable over time, although there are other researchers who believe it is possible for self-efficacy to change over time (Tschannen-Moran & Woolfolk Hoy, 2001), and still other researchers believe that there is a complicated inverse relationship between mathematical content knowledge, and teaching self-efficacy specifically (Stevens et al., 2013). It is possible that the seven-week period of the study was not sufficient for much change to self-efficacy. The main focus of the professional development was not to develop self-efficacy, but rather to appreciate content knowledge with the hope that increased self-efficacy would result. A change such as this could require more time.

Bandura (1997) also identified four sources of self-efficacy: mastery experiences, vicarious experiences, social persuasion, and physiological and affective states. In the larger context of the PD conducted by CMAST, teachers do have time to practice mastery, as they are introduced to new ideas, choosing what to try, and trying it for two to three weeks before bringing student evidence of it to a PLC. They are typically taken on instructional rounds once or twice a year to see other teachers who have mastered certain instructional practices for some vicarious experiences. Social persuasion is engendered through the Teacher Leader, who helps inspire their department or grade level toward positive action. The physiological and affective states of the participating teachers are not something CMAST has given much overt thought to, and this may be a future area to be strengthened in the program. As this series of 2 PDs is situated within the larger context of CMAST PD offerings, it may be that changes in self-efficacy would be more obvious after teachers have had more time to build mastery and more explicit practice in their classrooms. Additionally, because most of the participants had been participating in CMAST PDs for some time before the study began, there may have been an increase in their self-efficacy that was not observed within this study's timeline. Unfortunately, there was no baseline taken when participants first began the program and, as such, it may be that self-efficacy shifted but was not captured within the study.

Intended Teaching Practice

The results of the exit interviews performed after the final PD indicated that what teachers had learned would have an impact on their teaching practice, although most of the teachers indicated pedagogical strategies specifically. Only one teacher indicated that the PD would have no impact on her, stating that she was new to the math department and was "still trying to process everything and still teach." While not part of the technical length of the study, in my professional role with CMAST as faculty, I was able to observe one of the participating teachers three months after the last PD, during a lesson on unit rate. One of her students explained that a ratio and a fraction were two different things. During the post-observation debrief, the teacher mentioned specifically that she had emphasized the difference with the students based on what she had learned during the first content PD.

While observations such as this would be the ideal way to measure impact from the PD on actual teaching practices, the Theory of Reasoned Action (TRA) states that attitudes toward a behavior predict whether or not a person will adopt that behavior (Fishbein, 1979), and the Theory of Planned Behavior (TPB) extends TRA by adding that perceived control over the performance of the behavior in question also predicts whether or not the behavior will be adopted (Ajzen, 1991; Montano & Kasprzyk, 2008). As such, these theories indicate the importance of measuring intended behaviors, which was captured in the exit interviews, because they are the most proximal predictor of behavior. The behavior in question here is an inquiry-based approach to teaching math, which is what was modeled in the pedagogy portions of both PD sessions, but built upon the foundation of solid mathematical content knowledge. As part of CMAST, teachers have a degree of control over when and how to adopt the approach, but the expectation is that the approach will be adopted, and administrators attend PD sessions to send the message that they are also supportive of this approach and the teachers' use of it. This may have contributed to the high rate of teachers indicating they would adopt the newly learned practices. Furthermore, the intention to embed more inquiry-based methods of teaching suggest that teachers were considering a more constructivist approach.

Constructivism and Mathematics Teaching

The National Council of Teachers of Mathematics (NCTM) has been calling for a more conceptual understanding of mathematics since 1989. The Standards for Mathematical Practice, a subset of the Common Core State Standards are:

> varieties of expertise that mathematics educators at all levels should seek to develop in their students…The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding it Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

The average textbook currently overemphasizes procedures and rote memorization (Polikoff, 2015). In order for students to be able to do all that is spelled out in the Standards for Mathematical Practice, teachers will have to look beyond textbooks. World-renowned Stanford mathematician Keith Devlin, in a 2014 interview with *Forbes*, talked about the math of the future and how it differs from the math of the past. He mentioned that machines can now do "all of the procedural mathematics faster and more accurately than any human," so people are going to have to focus on creative problem solving rather than straight calculation. He further stated that what we need now is a "deeper understanding of how and why… mathematics works," and that the problems for which we need solutions today are "messy" (Shapiro, n.d.). Textbook problems are not messy. Teachers will have to allow students to explore mathematical ideas, try solutions, make mistakes, learn from them, revise their ideas, and try again. This is what math outside of school looks like, and teaching math in a more constructivist way will allow for that process.

Equity in an increasingly diverse world should be another concern of math educators. Constructivism facilitates students constructing their own mathematical understanding (National Research Council, 1989). Both Constructivism and Sociocultural Theory emphasize social interaction as a crucial component to student understanding. Critical Theory honors the knowledge that students already bring to the classroom, and Constructivism helps students build new knowledge on top of that existing knowledge. These theories working together facilitate access for all learners, not just those who have been successful in a traditional mathematics program.

Limitations

As with all research studies, findings are limited by the study's design. While the findings explored the connection between a content-based PD and content knowledge, self-efficacy, and intended classroom behavior of middle school mathematics teachers, some limitations to the research design should be considered. The sample size of the study was small, limiting its generalizability. The participating teachers were selected purposefully and had pre-existing relationships with the researcher. The period of time over which the study took place was very short, occurring only over approximately seven weeks. The motivations to participate for each teacher could have also varied considerably and research indicates that teacher motivation to participate can have a significant impact on PD results (Kennedy, 2016). The exit interviews were recorded on iPads. This method was readily available and understood by the participants. Participants were in a room alone and asked to point the camera toward the ceiling and record only their voices to preserve some measure of anonymity, however a few actually filmed themselves. This may have impacted their responses as they may not have felt comfortable being honest as they were easily able to be identified.

For the content portion of the study, the tasks were based upon proportional reasoning but there was no requirement to set up proportions in order to solve the tasks. It is also possible that those who solved it algebraically did not learn any new content from the PDs, although only one teacher on each task used an algebraic approach, indicating that this might not have been much of a limitation.

Future Research

In light of these limitations, future research might improve the study by considering the following. First, the sample size could be increased, and the results could be separated by type of credential held by each teacher. Teachers could be selected randomly from a population of middle school teachers, although to control for motivation it would be best to either choose from volunteers, or choose from a group of teachers required to attend, thus ensuring that each group had similar motivation to participate. Adding a measure of motivation to participate to the study would also allow researchers to control for varying degrees of motivation.

Given that self-efficacy did not significantly change in this study, examining Bandura's (1997) four sources of self-efficacy—mastery experiences, vicarious experiences, social persuasion, and physiological and affective states—in greater depth may contribute to a deeper understanding of the relationship between content knowledge PD and teaching self-efficacy. Of

Bandura's four sources of self-efficacy, the CMAST PD design does not address the physiological and affective states of the participants. CMAST and other PDs offered to teachers might benefit from addressing this specifically if the intent is to positively impact self-efficacy. To measure affective states, exit interview questions can include inquiry about the participants' emotional states during the PD as an example. Additionally, it would be beneficial to stretch the study out over a longer period of time to more accurately explore a connection between content knowledge and self-efficacy. Future researchers may wish to observe lessons, collect artifacts, or interview students to assess what impact the PD has actually had on classroom practice.

Implications

In addition to future research, there are several implications from the current study. These implications address both theoretical considerations as well as practical and policy considerations.

Theoretical Implications

Self-efficacy. The study did not show any impact on teacher's self-efficacy toward teaching math. It is possible that Bandura is correct about self-efficacy remaining stable over time, and that no amount of PD will impact teacher self-efficacy. However, given that self-efficacy is defined as "beliefs in one's capabilities to organize and execute the courses of action required to produce given attainments" (Bandura, 1977, p. 3) and growth mindset is defined as believing your capabilities are not fixed but can in fact change and improve over time (Dweck, 2006), the two ideas are connected. Dweck's studies were able to change mindsets, so it is likely that self-efficacy can be changed as indicated by Tschannen-Moran and Woolfolk Hoy (2001). Bandura (1997) also gave four sources of self-efficacy: mastery experiences, vicarious experiences, social persuasion, and physiological and affective states. It is possible that self-efficacy cannot be impacted unless all four sources are addressed—a possible avenue for future research. It would also be worthwhile to explore whether a connection exists between the Theory of Reasoned Action and the Theory of Planned Behavior and self-efficacy. For example, is it more likely that people with higher self-efficacy will follow through on their planned behavior?

The connection between sociocultural theory, critical theory, and constructivism. Constructivism asks that students to “see their responsibility in the mathematics classroom . . . as making sense of, and communicating about, mathematics” (Clements & Battista, 1990, p.7). Sociocultural Theory (Vygotsky, 1978) maintains that learning is a social process and that social interaction is crucial in the development of cognition. Critical Theory (Freire, 1970) posits that knowledge should be built upon through discourse. All three theories require students to use their pre-existing knowledge to make sense of mathematics through communicating with others. Future research should consider these three theories together.

Implications for Practice

First**,** the findings of this study could have implications for middle school administrators, as they are expected to be instructional leaders at their school sites. These administrators should be aware of the gaps in teacher content knowledge and know what types of PD actually assist teachers in improving their content knowledge. Similarly, school districts or charter organizations need to be aware of gaps in teacher content knowledge and how that can affect teacher practices in the classroom, so that supports for those teachers can be planned. If a constructivist approach is to be adopted in math classrooms, parents will need to be partners with the schools so they can understand the necessity for why math instruction will look very different from what they remember. This study may help those in charge of spending PD funds to consider where those funds might best be spent.

Policy Implications

At the state level, policy makers need to consider requiring more, or different types of content knowledge, than what secondary math teachers currently possess. The teachers' reactions to having to do a math task each time were indicative of high levels of discomfort with actually doing math. Currently, math teachers are not being taught math in a way that supports the conceptual understanding they are required to develop in their students, so the way in which math teachers are trained will need to be changed. Federal guidelines for universities with teacher preparation programs will need to be addressed. Common Core emphasizes college and career readiness as the aim of the standards, but current university programs are not preparing teacher to teach mathematics in a way that supports the learning of those standards. There should also be more oversight as to how PD money is spent. The Every Student Succeeds Act gives clear guidelines for how PD should look if it is paid for with federal funding, but there is no instrument to measure whether the PD meeting those criteria was effective. There is an implication that effective PD impacts student achievement, but there is no consensus on how to measure that impact.

Recommendations

The most immediate change should be to the way in which CMAST PDs are designed. Each new group of participants should have baseline data collected at their first PD session in order to more accurately measure the impact of the program on their self-efficacy and content knowledge. The content connection needs to be more specific, and teachers should be asked to do math at each PD. There should be time for them to ask questions, and explore their own misconceptions, as well as seeing vertically how the content connects through grade levels and courses.

Any middle school with math teachers needs to offer PD that helps teachers strengthen their conceptual understanding of their content. Meanwhile, this PD should model how to adopt a more constructivist approach in the classroom through inquiry-based pedagogy. School districts, charter organizations, and educational preparation programs should have regular conversations about whether teachers are receiving enough content-based training. That the change in mean scores was the smallest in the single-subject group suggests that teachers with all other types of credentials would benefit most from content support. However, given the importance of vertical articulation in the Common Core standards, specific support regarding development of concepts would be useful to holders of all types of credentials.

The amount of variance in scores for both pre- and post-tasks was greatest in the multiple subject only group. Although that group was the largest, which would provide the opportunity for more variance, it would be worthwhile to continue to study teachers with varying credential levels teaching secondary math as they have different content support needs.

Conclusion

We live in a time of very rapid and substantial change. It is imperative that we periodically examine routines and procedures in light of whether they are serving their intended purpose. The way in which students have been taught mathematics has not changed substantially in over 100 years. It is time to ask whether the methods we have been using for so long are still preparing students for the problems they will face and the jobs they are likely to perform.

Our society is becoming increasingly diverse. It is estimated that by 2055 the United States will not have a single ethnic or racial majority (Cohn & Caumont, 2016). Past practices in mathematics teaching have limited access to higher math courses for females, the socioeconomically disadvantaged, and students of color. It is not only unjust to continue to do so, but also foolish given the future makeup of the United States. Marilyn Frankenstein (1983) noted that, "Knowledge of basic mathematics and statistics is an important part of gaining real, popular, democratic control over the economic, political and social structures of our society" (). Degrees in fields such as engineering, computer science, and mathematics give the best financial return for the investment and give higher earnings overall ("Revenge of the Nerds," 2015). Enabling poor students and students of color access to math courses that put them on the path to earning those degrees will be essential for economic liberation. Thomas Jefferson (1778) said:

> experience hath shewn, that even under the best forms, those entrusted with power have, in time, and by slow operations, perverted it into tyranny; and it is believed that the most effectual means of preventing this would be, to illuminate as far as practicable, the minds of the people at large.

We owe it to all students to provide an education that enables them to be active participants in our democracy.

APPENDIX A

ACE'M Problem Solving Protocol

Doing & Thinking Through Mathematics

APPROACH

Ask Questions

When you first tried the task, did you…?

- ☐ Annotate the written representation of the task and recall connected facts
- ☐ Write out questions that come to mind about what properties the object, phenomena, or relationship has
- ☐ Create alternative representations (sketches, graphs, verbal descriptions)
- ☐ State the goal
- ☐ Identify the given relevant information and facts that might connect

When you were first thinking about the math, did you…?

- ☐ Ask if any of the given information is irrelevant/ missing/ contradictory
- ☐ Ask how I know if my representations and given information are relevant
- ☐ Identify questions that need to be answered to clear up points of confusion
- ☐ Ask if there are different ways to understand the task
- ☐ Contrast the current problem/task to previous ones
- ☐ Consider alternative ways of interpreting the task and/ or goal
- ☐ Ask if there is an error in my facts and approach
- ☐ Ask if I need to take a step back and work more on exercises before tackling this problem

CREATE A PLAN

Imagine a Pathway

Looking at your plan, did you…?

- ☐ Use the givens, identify models, relationships, and/or concepts that might apply
- ☐ Symbolically relate the givens with the model
- ☐ Using the model/concepts/relationships, establish connections and intermediate steps between givens and goal

Thinking through your plan, did you…?

- ☐ Decide if any approximations or assumptions, which weren't stated in the task, need to be made
- ☐ Decide if the model(s) are compatible with each other
- ☐ Compare the plan to others used in previous tasks to determine if reasonable
- ☐ Ask if all objects that look like this have the same property
- ☐ Evaluate the plan for completeness.
- ☐ Consider alternative plans
- ☐ Ask if I can describe the relationships
- ☐ Ask if there is an error in concept knowledge

EXECUTION
Create and Improve

When executing your plan, did you…?

- ☐ Follow the plan (solution pathway) until your goal is attained

When thinking through your execution, did you…?

- ☐ Continuously examine the plan to ensure it is working
- ☐ Ask if there is an error in the procedure
- ☐ Examine the outcome of the plan for coherence
- ☐ Compare the result of the plan against previous experiences to test if reasonable
- ☐ Check to see if the task has been completed
- ☐ Consider if the solution pathway will always work in all cases

APPENDIX B

SVMI
SILICON VALLEY
MATHEMATICS INITIATIVE

October 26, 2017

To Whom It May Concern:

Wendy Creek, doctorial candidate from LMU, has permission to use the following MARS performance assessment tasks, accompanied rubrics and Tools for Teachers documents; Lawn Mowing Grade 7 2005 and Photographs Grade 7 2006 in her doctorial studies. This include the right to publish these document as part of her study of her findings.

Sincerely,

Executive Director

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